Relationship between Trading Volume and Security Prices and Returns

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Abstract

The relationship between trading volume and securities prices is a complex one which, when understood properly, can lead to many insights in portfolio theory. Over the past forty years, much work has been done trying to understand this relationship. In this document, we will attempt to introduce and discuss some of these papers. First, we introduce basic topics of finance theory, such as the Capital Asset Pricing Model and two-fund separation. With this knowledge, we proceed to discuss how volume and price move together, how unusual volume can be a predictive measure of future price changes, and also how volume can allow us to infer a hedging portfolio. In each case, we present theoretical models which support empirical results. Finally, we analyze some sample price and volume data around the most recent quarter of earnings announcements.

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1 Introduction and Motivation

When investors today read the business section of the paper, or obtain online quotations of their favorite stocks, one of the statistics which usually goes unnoticed is volume data. After seeing the price of a security, which is usually of primary interest, an investor may look next at data such as yield, price-to-earnings ratio, market capitalization, or ex-dividend date, before even considering the volume statistic. Despite being ignored by many investors, trading volume does have a relationship to price data, returns, and other aspects of portfolio theory.

The following report provides an overview of research in the area of trading volume within the wider discipline of stochastic finance. The analysis of trading volume and its relationship with security prices and changes in price is a topic that has been considered for over 40 years. Its roots are generally credited to the work of Osborne [36]. In his seminal work, he modeled price changes according to a diffusion process that had a variance dependent on the quantity of transactions on that particular issue. With this, he began a long line of work that considered the possible relationship between returns and the volume of trading. Before these works are discussed and analyzed, we shall try to motivate research in this area and hopefully answer the question *"Why is the relationship between price and volume of any interest?"*

1.1 Gazing into the Crystal Ball - Predicting Price Movements

One of the most sought after results of financial economics research is the predictability of asset prices. As the reader might expect, one aspect of financial economics involves modeling future prices of equities, bonds, or other derivative securities. Early research focused on attemping to predict future prices based on historical prices alone.

1.1.1 Martingales

In 1565, Girolamo Cardano wrote in his text *Liber de Ludo Aleae* (The Book of Games of Chance) that in a "fair game" the total winnings, represented by P_t, is a stochastic process satisfying the following condition:

$$E(P_{t+1}|P_t, P_{t-1}, ..., P_0) = P_t$$

meaning that the best estimate of what you will have tomorrow is what you have today. This process is often called a *martingale* [5]. Exactly four hundred years later, Samuelson [40] showed that efficient markets exhibit this behavior and described this condition as *weak-form market efficiency.* The Efficient Market Hypothesis, introduced by Fama [12], states that in an efficient market there are a large number of "rational profit-maximizers" that actively compete, each trying to predict future market values. The interaction of these participants causes the current price to fully reflect the expectation of the future price of the security. In essence, the more efficient a market, the more unpredictable future pricing will be, with the expected value of future prices equal to the current price, as indicated by the equation above. A reader at this point may argue that historical equity returns in the domestic market have been positive over the long run, thus making this hypothesis questionable. In fact, it was shown in the 70's [20, 24] that the efficient market hypothesis does not hold for asset prices, but with the proper adjustment for risk and the prevailing risk-free rate [8, 16, 20], Cardano's equation does hold and weak-form market efficiency exists. From seeing that price alone was insufficient to predict future prices, researchers sought other factors to aid in their analysis[1].

[1] It should be noted that recent research has shown that prices do not follow a purely random walk [26], but the point of this discussion is to motivate why researchers have considered alternative factors, such as trading volume, to predict future price trends.

1.1.2 Trading Volume

One factor many have considered in the prediction of prices is trading volume. Volume is a measure of the quantity of shares that change owners for a given security. For instance, on the New York Stock Exchange (NYSE), the average daily volume for 2002 was 1.441 billion shares, contributing to 40.9 billion dollars of securities traded each day among the roughly 2800 companies listed on the NYSE [18]. The amount of daily volume on a security can fluctuate on any given day depending on the amount of new information available about the company, whether options contracts are set to expire soon[2], whether the trading day is a full or half day, and many other possible factors. Of the many different elements affecting trading volume, the one which correlates the most to the fundamental valuation of the security is the new information provided. This information can be a press release or a regular earnings announcement provided by the company, or it can be a third party communication, such as a court ruling or a release by a regulatory agency pertaining to the company. For example, McDonald's Corporation (NYSE:MCD) has an average trading volume of 7.58 million shares per day. On December 17, 2002, they announced a warning and reduction of expected earnings. The news led to trading of 35.17 million shares that day, about five times the average, and a drop in price of 8%. The abnormally large volume was due to differences in the investor's view of the valuation after incorporating the new information. Because of what can be inferred from abnormal trading volume, the analysis of trading volume and associated price changes corresponding to informational releases has been of much interest to researchers.

Returning to the question of, why consider trading volume and its relationship to prices, Karpoff [22] suggests the following four possible reasons. First, it adds insight to the structure of financial markets. The correlations which are found can provide information regarding rate of information flow in the marketplace, the extent that prices reflect public information, the market size, and the existence of short sales and other market constraints. Second, studies that use a combination of price and volume data to draw inferences need to properly understand this relationship. For example, trading volume is often used to determine whether or not a price change was due to any informational content, and also whether investor interpretations of information are consistent or differing. Some researchers [38] have used volume and price changes to determine that shareholders hold securities primarily because of dividend yields. Beaver [2] asserts that the volume corresponding to a price change due to new information indicates how much investors differ in the interpretation of the new data. As one can imagine, the validity of many of these inferences rely on the relationships between price and volume.

Third, understanding the price-volume relationship in futures and other speculative markets is vital for one to determine why the distribution of rates of return appear kurtotic[3]. One theory is that rates of return are characterized by a class of distributions with infinite variance, known as the stable Paretian hypothesis. Another theory is that the data comes from a mixture of distributions which each have different conditional variances, known as the mixture distribution hypothesis. Research has shown that price data is generated by a stochastic process with changing variances which can be predicted or estimated by volume data. These price/volume analyses support the mixture of distribution hypothesis in the following way. If we measure changes in volume data as a proxy for variances at different events, we observe that the distribution of returns exhibit different conditional variances. As a result, our return data follows a mixture of distributions.

Fourth, price variability affects trading volume in futures contracts. This interaction determines whether speculation is a stabilizing or destabilizing factor on futures prices. The time to delivery

[2]Options contracts expire on the third Friday of each month, unless that date is a holiday.

[3]The degree of peakedness of a distribution. It is a normalized form of the fourth central moment of a distribution [14].

of a futures contract affects the volume of trading, and possibly also the price.

1.1.3 Interpretation of Information

As Beaver noted, volume is a useful tool in determining how much disagreement exists with the arrival of new information. Anything that causes investors to act can be described as information, whether or not it truly has any fundamental impact on the underlying valuation of the company. For example, a University of Michigan study found that, "in the absence of clear financial information, investor decisions are swayed by the aesthetics of financial reports" [42]. It is doubtful that there exists any significant correlation between the aesthetics of a company report and its future earnings, yet the study shows that some individuals attribute value to an organization which produces an aesthetically pleasing report.

Sometimes, information on a company can impact the volume and price of another unrelated company due to the sheer similarity of the ticker symbol. In particular, Rashes [37] discussed an example where information releases on MCI Communications (Nasdaq:MCIC[4]) led to increased volume on Massmutual Corporate Investors (NYSE:MCI), a case of co-movement due to ticker confusion. MCI Communications was a large telecommunications firm that was acquired for more than 20 billion dollars, while Massmutual Corporate Investors is a closed-end fund which trades with net assets of roughly $200 million. Rashes found that Massmutual's top volume days between 11/1/1996 and 11/13/1997 all occurred on days when there was merger news on MCI Communications, showing that Massmutual's volume was correlated with MCI Communications' trading volume, but not those of other telecommunications companies. The latter is certainly to be expected because Massmutual, during that period, did not hold any major telecommunications company stock, while the former can only be attributed to investor confusion.

1.2 Initial Work

Following is a summary of some of the initial work in this area of discipline. First, we analyze Osborne's seminal work, and then briefly discuss a work by Ying.

1.2.1 Volume's Effect on Variability of Returns

In 1959, Osborne [36] hypothesized that securities prices could be modeled as a lognormal distribution with the variance term dependent on the trading volume. In particular, if $y(\tau) = ln(\frac{P(t+\tau)}{P(t)})$, where $P(t+\tau)$ and $P(t)$ are the price of some issue at times $t+\tau$ and t, respectively, then the steady state distribution of y may be expressed as

$$\phi(y) = \frac{1}{\sqrt{2\pi\sigma^2\tau}} e^{\frac{-y^2}{2\sigma^2\tau}},$$

where σ is the dispersion that is positively correlated to the amount of trading volume. By verifying that this model held empirically, Osborne concluded that the log return process was a Brownian motion process. Furthermore, if we examine the probability distribution of price itself, we see that

$$f_P(P) = \phi(y = ln(\frac{P}{P_0}))\frac{dY}{dP} = \frac{1}{\sqrt{2\pi\sigma^2\tau}} exp(-\frac{ln(\frac{P}{P_0})^2}{2\sigma^2\tau})\frac{1}{P}.$$

When we compute the expectation, we obtain

$$E(P) = \int_{p=0}^{\infty} P f_P(P) dP = P_0 e^{\sigma^2\tau/2} \approx P_0(1 + \frac{\sigma^2\tau}{2})$$

[4] MCI merged with WorldCom, Inc.; so, this ticker is no longer active.

through a change of variables, completion of square, and Taylor series approximation. From the empirically calculated values of σ, Osborne was able to predict a 3 to 5% annual return from this model.

The fact that greater activity on a security will produce more variance in the price may seem reasonable, even intuitive. However, what was remarkable was the fact that the annual drift predicted by this Brownian motion process could partially explain the annual returns that the market actually bore.

1.2.2 Volume's Predictive Nature for Price Changes

Seven years later, in 1966, Ying produced a paper [43] which applied a series of statistical tests to a six-year daily series of price and volume. As is the case for most of the other analyses discussed in this document, Ying normalized the trading volume by the number of shares outstanding to avoid any biases from issues with larger number of outstanding shares[5]. Similarly, prices were adjusted to reflect quarterly dividends.

To apply the test of his hypotheses, Ying used as volume data the New York Stock Exchange (NYSE)[6] daily percentage volume (also called turnover), and Standard and Poor's 500 index returns from January 1957 to December 1962 for price data. Critics to his research have argued that the underlying issues for volume and price data were not exactly the same, as well as the fact that his adjustments for price were over quarterly dividend data, while the daily volume data was adjusted by monthly total share data. Furthermore, it was found that some of these conclusions were inconsistent with weak form of market efficiency[7], although this fact alone might somewhat be expected, as any relationship that can be found between volume and future prices will reject the weak form of market efficiency.

Details of his analysis may be found in Appendix B.2, but his main conclusions were

- A small volume is usually accompanied by a fall in price
- A large volume is usually accompanied by a rise in price
- A large increase in volume is usually accompanied by a large price change.
- A large volume is usually followed by a rise in price
- If the volume has decreased (increased) five straight trading days, the price will tend to fall (rise) over the next four trading days.

1.3 Document Outline

The previous two examples illustrate the origins of research in trading volume. In this report, we will summarize and analyze three more recent papers in this field [4,22,28], and the author will then provide some new analysis on the serial relationship between volume and price changes around 4th quarter 2002 earnings announcements on the thirty stocks that comprise the Dow Jones Industrial Average. This paper will be outlined as follows:

Basic Finance Background – Section 2 will provide a quick summary of some of the main concepts of basic portfolio theory.

[5] Volume divided by Shares Outstanding is called turnover, and is often a preferred indicator for analysis.

[6] Most of the volume in domestic trading then occurred on the NYSE; the NASDAQ did not exist until 1971.

[7] The weak form of market efficiency states that stock prices reflect all information that can be found from historical prices, trading volume, and short interest. Mathematically, one would say that the price satisfies a Markov process.

Volume-Price Relationship – Section 3 will look at how volume and price move together and will present a probabilistic model for trading.

Serial Correlation of Returns – Section 4 will consider how abnormal trading volume affects the subsequent returns on the market.

Inferring the Hedging Portfolio – Section 5 will examine how we can use volume data to infer the hedging portfolio.

Analysis of Current Market Data – Section 6 will discuss what kind of relationship exists between price and volume in our current market.

Conclusion – Section 7 will conclude with a summary of this document. The appendix has background on the mathematics and statistics required for this paper.

2 Overview of Portfolio Theory

During the analysis of current finance papers, some basic concepts will be referenced. This section provides a quick overview of portfolio theory and some financial terms of interest. A reader knowledgeable on the topic of finance may skip this section without any loss of continuity.

2.1 Capital Asset Pricing Model

The Capital Asset Pricing Model (CAPM) was developed by Sharpe, Lintner, and Mossin [25, 34, 41]. Despite its restrictive constraints and assumptions, the model provides a general idea of the relationship between the expected return of an asset and the riskiness of it. Details of CAPM follow.

For CAPM to hold, the following assumptions which basically generalize investors as a homogeneous group of individuals must be true [3]. Namely,

- no single investor has a significant portion of the total wealth in the world, thus making all investors price-takers who do not affect the price of securities through their own actions.
- we have a single-period horizon
- investments are limited to publicly traded financial assets and risk-free borrowing or lending. This eliminates the investments of human capital, private companies, and government-funded projects.
- there are no taxes or transaction costs
- all investors are rational, investing according to Markowitz [30]
- all investors have the same economic expectations of the world and its future financial development

If these six conditions hold, the following can be concluded:

- every person will hold a market-weighted portion of each asset in their portfolio
- the market portfolio will be on the efficient frontier
- the risk premium on the market portfolio will be proportional to its risk and the amount of risk aversion of a representative investor
- the risk premium on each individual asset will be proportional to the risk premium on the market portfolio and the beta of that security.

2.1.1 Why Everyone Holds the Market Portfolio

At first glance, it might not be clear why everyone chooses to hold the market portfolio. If everyone is rational, has the same investment options and time horizon, and possesses the same future economic expectations, then each person will have a portfolio which is a linear combination of the risk-free bond and the same market portfolio (depending on the amount of risk each individual wants to bear). Now that we know that all investors will hold the same market portfolio, what guarantees us that every stock will be in the portfolio. Suppose a certain stock does not belong in the portfolio. That is, no one wants to buy the security. By the principle of supply and demand, the price of this stock will drop until it becomes an attractive investment. Once it reaches this point, it will become included in the market portfolio.

Since every person holds the market portfolio, when aggregated across the entire market, the total holdings of each security will equal the market capitalization of that security. Thus, the percentage of each security in the market portfolio is proportional to its overall market capitalization.

2.1.2 Beta of a Security

Beta is a measure of how correlated the stock is with the market portfolio. Mathematically,

$$\beta_i = \frac{Cov(r_i, r_m)}{\sigma_m^2},$$

where r_i, r_m are random variables representing the returns of the particular stock, and the market portfolio, respectively, and σ_m^2 is the variance of the market return. If r_f is the return of the risk-free asset, then

$$E(r_i) - r_f = \beta_i E(r_m) - r_f.$$

It is assumed that r_f is a fixed quantity that represents the risk-free interest rate.

To better understand this, suppose that the market returns 9% annually, and the prevailing risk-free rate is 3%. A security with a β of 0 is not correlated to the market, and will have an expected return equal to the risk-free rate of 3%. A security with a β of 2 varies twice as much as the market, but has an expected return of 3+2(9-3)=15%.

2.1.3 Two-fund separation

One of the terms we will use throughout this paper is two-fund separation. The two-fund separation theorem [27] is a theorem which asserts that all investors will choose to invest in only two different funds, and no others. In the case described above, the two funds are the risk-free bond and the market portfolio. From the previous subsection, we see that individuals who choose to invest in the stock market will want to hold the market portfolio. However, as we will see in Section 2.3, the market portfolio may pose too great of a risk for some investors. As a result, they will adjust their market holdings according to their penchant for risk, and will invest the remaining amount in the risk-free bond.

A useful fact of two-fund separation is that share turnover will be identical for all securities. This is because an individual who chooses to adjust his portfolio will buy or sell the same proportion of each stock in order to maintain the correct balance in his market portfolio.

We could also choose to apply a (K+1)-fund separation theorem, for arbitrary values of K. When we do this, we will find that turnover will satisfy an approximately K-factor linear structure. For a discussion of multiple factor linear structures, we shall introduce a model which allows for this called the Arbitrage Pricing Theory.

2.2 Arbitrage Pricing Theory

The Arbitrage Pricing Theory (APT) was developed by Ross [39] and is a more generalized pricing model than CAPM. With APT, we can have multiple risk factors affecting price[8]. Ross asserted that the return of individual securities could be separated into systematic and non-systematic risk. Systematic risks include many possible macroeconomic factors in the market. Non-systematic risk measures the risk of the individual issue, independent of other securities and market factors.

Mathematically, if a particular security i had a single factor risk, then its return could be modeled as

$$r_i = E(r_i) + \beta_i F + e_i,$$

[8] Recall for CAPM, there is only one factor which is that of the market.

where F is the deviation of the factor from its expected value (hence, zero-mean by construction), β_i measures the sensitivity of this factor to the security, and e_i is a zero-mean observation of the non-systematic risk of the issue, uncorrelated with F. e_i is zero-mean because the first term absorbs the expectation return, with individual security distubances considered. The corresponding variance of r_i is

$$\sigma_i^2 = \beta_i^2 \sigma_F^2 + \sigma_{e_i}^2,$$

where σ_F^2 is the variance of F. Regression techniques are used to solve for the β_i constants. Appendix A.6 explains how this is done.

2.2.1 Using APT to Justify Diversification

As an example, this model of the return of individual securities can be used to show why it is beneficial to have a well-diversified portfolio. Suppose there are N such securities in the market, each with a single risk factor F. If we equally weight the N securities in our portfolio, then the return would be

$$r_N = E(r_N) + \beta_N F + e_N,$$

where $\beta_N = \frac{1}{N}\sum_{i=1}^{N} \beta_i$ and $e_N = \frac{1}{N}\sum_{i=1}^{N} e_i$. Since it is assumed that individual non-systematic risks are independent of each other and the economic risk factor, the variance of the portfolio is

$$\sigma_N^2 = \beta_N^2 \sigma_F^2 + \sigma_{e_N}^2,$$

where $\sigma_{e_N}^2 = \sum_{i=1}^{N} \frac{1}{N^2}\sigma_i^2$. If we define $\sigma_{avg}^2 = \sum_{i=1}^{N} \frac{1}{N}\sigma_i^2$, then we see that $\sigma_{e_N^2} = \frac{1}{N}\sigma_{avg}^2$. So, with N equally weighted securities in a portfolio, the non-systematic variance is N times less than the average variance of each of the individual variances. Thus, with diversification, risk is effectively reduced. As a result, all investors, like in the CAPM, will choose to hold a portfolio of assets rather than any individual security. Unlike CAPM, we can consider multiple factors in our model.

2.2.2 Multi-factor Models

Multifactor models are often used to relate one financial statistic to multiple factors in the economy, or multiple events, either specific to the firm or general to the entire market. The equations are similiar to the single factor models, except that additional factors with coefficients are added linearly to the equation. For example, in our discussion of volume, one might speculate that volume is higher on days of information release, and also on days when the federal reserve adjusts interest rates. So, one would write

$$V_i = E(V_i) + \beta_I I + \beta_R R + e_i,$$

where I is either 1 or 0 depending on whether the company released information, and R is 1 or 0 depending on whether interest rates were adjusted.

2.3 Efficient Frontier and Markowitz's Portfolio Selection Model

Suppose we have a portfolio of securities, and we consider all possible combinations of these assets. If we create a plot of expected return versus standard deviation of th all different possible combinations of securities, one will see that for every expected return value, there exists a minimum standard deviation. The curve generated by the collection of such points is called the minimum-variance frontier. At some point, the curve reaches its minimum standard deviation. This point is called the global minimum-variance portfolio. All points with smaller expected returns (and correspondingly larger standard deviations) would be inefficient portfolios. So, if we remove these

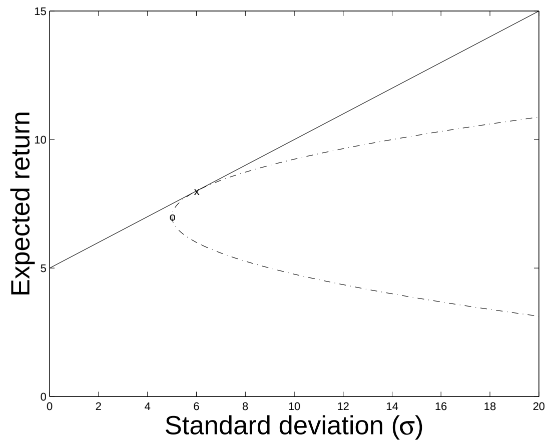

Figure 1. A minimum-variance frontier (dashed-dotted curve) with capital market line tangent to the frontier at the location of the market portfolio (x). The global minimum-variance portfolio having expected return of 7% and standard deviation of 5% is also shown (o). A risk-free rate of 5% is assumed.

points from consideration, the global minimum-variance portfolio and all points on the minimum-variance frontier with larger expected values create the efficient frontier. Harry Markowitz's work in 1952 [30] identified this region as an efficient set of portfolios[9].

Figure 1 shows the graph of the minimum-variance frontier. The optimal capital allocation line is the line which starts on the y-axis at the risk-free rate, and is tangent to the efficient frontier. It coincides with the capital market line. The point of tangency is the market portfolio discussed earlier in the capital asset pricing model (Figure1 shows this graphically).

The reason investors will invest in a linear combination of the market portfolio and the risk-free asset can be seen from Figure 1. At every level of risk, the point on the capital market line has a higher expected value than any point on the efficient frontier.

For example, suppose the risk-free rate is 5% for our figure above, which has a minimum-variance portfolio having standard deviation of 5% at an expected return of 7%. If we draw a straight line which is tangent to the efficient frontier, its point of tangency will be at 6% standard deviation and 8% return, which represents our market portfolio. This line is known as the capital market line, and all investors under CAPM will have their portfolio somewhere along this line, depending on the amount of risk they are willing to bear. For a person who is willing to take 3% standard deviation of risk, he can achieve this by investing half of his money in the risk-free asset, and the other half in the market portfolio. His expected return will be 6.5%. In general, the expected percentage return for an investor investing along the capital market line in this particular example can be written as

$$E(r) = 5 + \frac{1}{2}\sigma.$$

■ 2.4 Short Selling

A short sale is a method for an investor to bet against the prospects of a company. Instead of purchasing shares of a security, the investor "borrows" a share from a broker and sells it. Eventually, the short-seller must purchase the stock back to return it to the broker. If the stock drops in value as the investor hoped, then the short sale will lead to profit. While the investor is borrowing the

[9] Markowitz received the Nobel Prize in economics in 1990 for his contributions to economics.

stock, he must pay any dividends to the broker.

While the investor has a short position, he has borrowed someone else's shares. The investor's broker usually arbitrarily assigns another shareholder as the person who is loaning the security. However, if this individual opts to sell the stock, the broker will merely reassign who is loaning the stock to the short-seller.

Short selling poses much greater risks than the standard procedure of buying, or going long on stock. Unlike an investor in a security, the short-seller has unlimited liability. For example, if an investor shorts a stock at $100 per share and the stock triples, if the investor buys the share back then to cut his losses, he will have lost $200 on that one share. Because these risks exist, brokerage firms usually restrict the type of clients who are allowed to short-sell securities.

In addition, exchange rules allow short sales only on an uptick, meaning a time where the last trade led to an increase in recorded price change. This is to prevent a snowball effect during a selloff as prices, already falling quickly, would receive more downward pressure from the additional supply of sellers. The existence of this rule and the added restrictions by brokerage firms makes short-selling not as easy as standard investing, and we will see that there are market effects due to this asymmetry.

2.5 Lemons Principle

George Akerlof, a winner of the Nobel Prize in Economics in 2001 for his work on the information asymmetry in the sale of used cars, first introduced the Lemons Principle in 1970 [1]. In this landmark paper, Akerlof discussed how the existence of goods with many quality grades leads to problems in the theory of open markets because of the idea of adverse selection. For example, a person trying to sell a used car has information that prospective buyers do not have and cannot glean from a limited amount of time examining and inspecting the vehicle. The seller has potentially years of experience that he cannot fully disclosure, even with his best effort, regarding different aspects of the vehicle.

So, this fact results in two things. First, Akerlof claimed that this explains why new cars depreciate as one drives it away from the dealership, not any other reason. If the buyer decides to sell it the next day, the implication is that the buyer has obtained new information about the car to warrant trying to sell it. The second, related result is that when a person sells a used car, the initial price set can contain informational content, namely its true condition. If the initial price is set too low, even for the primary purpose of selling it quickly, buyers may believe that something not clearly visible is wrong with the vehicle. A similar argument can be made for homes placed for sale.

For our purposes, the importance of the Lemons Principle is that pricing and other factors, such as time on the market, or number of times a product has changed hands, can provide informational content to individuals who have not been directly informed of any new information.

Now that we have provided a basic discussion of portfolio theory and other concepts which will be useful to know throughout the remainder of the document, we now proceed to the discussion of recent papers on trading volume.

3 The Volume-Price Relationship

This section focuses on Karpoff's work [22] and related papers that contribute to the analysis. Following will be a discussion of their findings followed by comments. Karpoff's focus is how, if in any manner, do prices and volume move together?

3.1 Volume is Positively Correlated with Absolute Price Changes

A Wall Street adage says *"It takes volume to make prices move."* As discussed in Section 1.2.1 of initial work, researchers hypothesized long ago [36] that volume would drive variability, and was subsequently supported by many empirical studies[10]. These works include Crouch, who found a correlation for market indices and individual stocks using daily price and volume data. Clark determined this relationship to be true in cotton futures markets for daily data. Morgan used four-day intervals and monthly data from 51 different individual stocks and found the same conclusion. Twelve other authors verified this statement using different intervals and different securities. The net conclusion was that the absolute price and volume correlation existed for both equity and futures markets across all time intervals, although the correlation was often weak, especially with transactions data. The weakness in correlation, however, can be attributed to the fact that short selling is often more difficult than buying a stock. This asymmetry causes lower volume in accordance with price reductions, and can be seen in the following theoretical trading model.

3.2 Probabilistic Model for Trading

Copeland introduced [6,7] a probabilistic model that explains and supports the hypothesis that the maximum volume occurs when there is a consensus about new information, and with consensus we have a maximum price change in one direction or the other. Suppose we consider a world where there is one single indivisible asset which is traded among N market participants. Initially, one person owns the security and all participants have the same information about the asset, thus sharing the same valuation for the security. Now, suppose that new information is disseminated sequentially one investor at a time, with each investor classifying the new news as positive or negative. If a newly-informed individual is the first to view the information as positive and does not own the stock, he will obtain it. He can obtain it because the owner of the stock is either uninformed or has a negative view on the issue; so, the owner will be willing to sell it to someone who has a stronger belief regarding the value of the asset. The assumption is that the owner will not gain any information based on the buyer's interest in the asset. As a result, he will not gain any additional attachment or interest in the asset that could hinder the likelihood of the transaction. The reasoning is that the owner assumes that the two traders simply have different utility curves. On the other hand, if an informed individual considers the new news to be bad, then he will sell the stock if he owns it. If he does not own the stock, he will do nothing[11]. The third case involves an individual who is positive about the asset, but is not the first to have such an opinion. Since we assume that each informed person will value the asset independently of other informed individuals, the probability that this person will value the asset more than every previous R positive informed investors is $\frac{1}{R+1}$. In this scenario, he will purchase the asset; otherwise, no transaction will occur.

3.2.1 Consensus on Interpretation of Information

Consider initially the case where all traders will be optimistic about the stock when they receive information. Since the value each trader will have on the stock is random, the probability that the

[10] The references for all of these works may be found in Karpoff's paper.
[11] A case where short-selling constraints can affect trading volume.

m^{th} trader will buy the asset is $\frac{1}{m}$. We do not have to adjust for the case of owning the asset since the first trader will buy it for sure; so, none of the following traders will own it when they receive new information. Let the random variable B_m be 1 when the m^{th} trader will trade to purchase the asset, and 0 if he does not make a purchase. To determine the trading volume in our model, we need to find the expected value of the sum of $B_1, B_2, B_3, \ldots, B_n$. Start with the first person informed of the new information. This trader will place a trade unless he owns the security; so,

$$Pr(B_1 = 1) = 1 - \frac{1}{N},$$

and correspondingly

$$Pr(B_1 = 0) = \frac{1}{N}.$$

Thus, the expected value and variance of this binomial random variable are

$$E(B_1) = 1 - \frac{1}{N}$$

and

$$\sigma^2 = \frac{1}{N} - \frac{1}{N^2}.$$

For m > 1, as discussed above,

$$Pr(B_m = 1) = \frac{1}{m}.$$

If we take the z-transform[12] of this random variable, we obtain

$$p^T_{B_m}(z) = E(z^{B_m}) = 1 - \frac{1}{m} + \frac{z}{m}.$$

The z-transform of the sum of two independent random variables is the product of the z-transforms of the individual random variables (a proof can be found in Appendix A.1). Since the random variables are pairwise independent, the z-transform of the entire sum is just the product of each of the z-transforms. Thus, if

$$S = \sum_{m=2}^{N} B_m$$

then the z-transform of the sum is

$$p^T_S(z) = \prod_{m=2}^{N} (1 - \frac{1}{m} + \frac{z}{m}).$$

Now we can compute the derivatives of $p^T_S(z)$ to aid us in determining the first and second moments of S.

$$\frac{d}{dz} p^T_S(z) = \sum_{m=2}^{N} \frac{1}{m} \prod_{n \neq m} (1 - \frac{1}{n} + \frac{z}{n}).$$

Taking another derivative, we obtain

$$\frac{d^2}{dz^2} p^T_S(z) = \sum_{m=2}^{N} \sum_{n \neq m} \frac{2}{mn} \prod_{r \neq m, r \neq n} (1 - \frac{1}{r} + \frac{z}{r}).$$

[12]For this case, we can arrive at the expected value in a much simpler way, but we need this to be able to calculate the variance.

N	E(T)	σ_T	CV(T)
2	1.000	0.707	0.707
5	2.083	0.990	0.475
10	2.829	1.212	0.429
50	4.479	1.701	0.380
100	5.177	1.887	0.365

Table 1. Expected number of trades, standard deviation, and coefficient of variation (CV(T)) for different number of investors/traders.

From this, we can compute the expected value and variance, using identities derived in Appendix A.1.

$$E(S) = (\frac{d}{dz}p_S^T(z))|_{z=1} = \sum_{m=2}^{N} \frac{1}{m}$$

and

$$\sigma_S^2 = (\frac{d^2}{dz^2}p_S^T(z))|_{z=1} + (\frac{d}{dz}p_S^T(z))|_{z=1} - [(\frac{d}{dz}p_S^T(z))|_{z=1}]^2$$

$$= 2\sum_{m=2}^{N}\sum_{n\neq m}\frac{1}{mn} + \sum_{m=2}^{N}\frac{1}{m} - \sum_{m=2}^{N}\frac{1}{m^2} - 2\sum_{m=2}^{N}\sum_{n\neq m}\frac{1}{mn}$$

$$= \sum_{m=2}^{N}\frac{1}{m} - \sum_{m=2}^{N}\frac{1}{m^2}$$

So, the expected number of trades T is

$$E(T) = E(B_1 + S) = E(B_1) + E(S) = 1 - \frac{1}{N} + \sum_{m=2}^{N}\frac{1}{m}.$$

Similarly, by independence, the variance of T is

$$\sigma_T^2 = \frac{1}{N} - \frac{1}{N^2} + \sum_{m=2}^{N}\frac{1}{m} - \sum_{m=2}^{N}\frac{1}{m^2}.$$

Defining the coefficient of variation as the standard deviation divided by the expected value ($\frac{\sigma_T}{E(T)}$), we see from Table 1 and Figure 2 that the coefficient of variation decreases as N increases. This fact is important in establishing that a specialist who is a market maker for a particular stock will benefit from less relative variability in trading volume if there are more participants in the market.

If one considers the opposite extreme where all of the traders are pessimists, and applying the fact that

$$\sum_{m=2}^{N}\frac{1}{m} = \sum_{m=1}^{N-1}\frac{1}{N-m+1},$$

one will reach the identical conclusion.

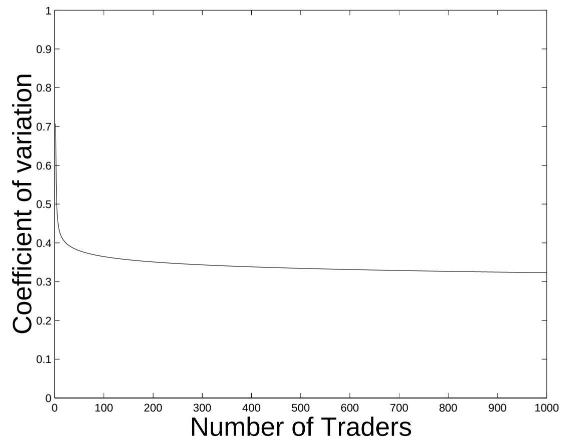

Figure 2. Coefficient of variation can be seen to be a decreasing function of N.

3.2.2 General Case of Information Interpretation

Now consider the case where some of the investors view the new information optimistically, while others pessimistically. Again suppose there are N traders, with p the fraction of optimists and q the fraction of pessimists, and each trader is either one or the other[13]. Now, a transaction can occur at the m^{th} trader in only one of the following three ways:

1. If all of the first (m-1) traders are pessimists and the m^{th} trader is a pessimist who also owns the asset, he will sell it to one of the investors who has yet to receive information[14].
2. If all of the first (m-1) traders are pessimists and the m^{th} trader is an optimist who does not own the asset, he will purchase the asset.
3. If there has been a prior optimist and the m^{th} trader is also an optimist, then he will purchase it if his valuation of the asset is larger than all prior optimists.

Mathematically, the probability that the m^{th} investor makes the purchase can be written as

$$Pr(B_m = 1) = (\frac{(qN)!}{(qN-m+1)!}\frac{(N-m+1)!}{N!})(1-\frac{pN}{N-m+1})(\frac{1}{N-m+1})+$$

$$(\frac{(qN)!}{(qN-m+1)!}\frac{(N-m+1)!}{N!})(\frac{pN}{N-m+1})(1-\frac{1}{N-m+1})+$$

$$\sum_{r=U}^{V}(\frac{\binom{m-1}{r}\binom{N-m+1}{pN-r}}{\binom{N}{pN}})(\frac{(pN-r)}{(N-m+1)})(\frac{1}{(r+1)}),$$

where the first line matches the first condition, the second line the second condition, and the third line the third condition described above, with $U = max(1, m-1-qN)$ and $V = min(pN-1, m-1)$.

[13] i.e. p + q = 1, p $\neq$ 0, q $\neq$ 0

[14] The case for m=N will not happen since this would require p = 0.

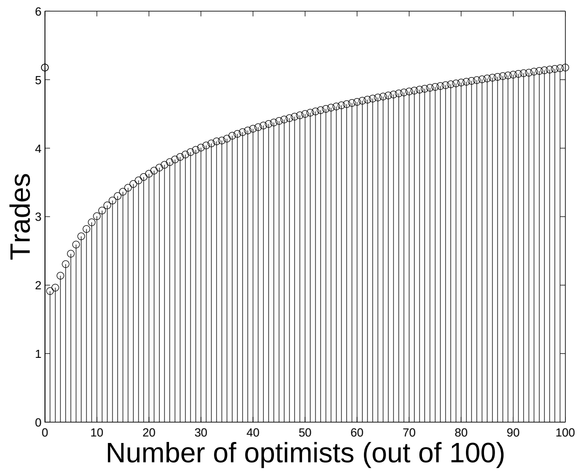

Figure 3. Number of trades as a function of optimists out of 100 traders. Note that this is an increasing function, which shows the bias which exists due to short-sales constraints. The code written to generate this graph may be found in Appendix B.3.

Again, we compute the expected number of trades by taking the z-transform's first derivative and evaluating it at $z = 1$, or more simply, the expected number of trades is the sum of the probability that each individual trader will want to make a trade[15]

$$E(T) = \sum_{m=1}^{N} Pr(B_m = 1).$$

Figure 3 shows that the number of trades for p > 0 is an increasing function, with a discontinuity at p = 0. The reason for the discontinuity is because as soon as there exists one optimist, there will only be one or no trades, depending on whether or not the optimist holds the asset to start with. The monotonic characteristic of the plot is a result of a short-sales constraint. In this simple case, the constraint is that if you do not hold the asset, you cannot sell it. Therefore, pessimists have less opportunity to act than optimists. Relating to price, the more optimists there are, the higher the final value of the asset after the information is transmitted to all traders, and when everyone is an optimist or a pessimist, we have the largest change in price, which occurs when we have the largest amount of volume, as seen from our calculations. This fact allows us to connect this model with the volume-absolute price change relationship.

Generalization to Divisible Assets

The result of this model can be generalized into markets with divisible assets (i.e. multiple shares). In this case, when optimists trade on new information, they drive the price upwards, causing uninformed participants to sell some or all of their stock, since every share purchased by an optimist had to be sourced from another shareholder. Once the pessimists are informed of the news, they will only be able to sell whatever shares they have remaining, and will not be able to sell any more stock to future optimists or pessimists that have a less bleak view of the new news.

Relationship between Trades and Trading Volume

In order for this model to be relevant to our discussion of trading volume, we must be sure that the

[15] By the linearity of the expectation operator.

number of trades is closely related to the trading volume (this may seem obvious, but any individual trade can involve any number of shares for a divisible asset). Copeland [7] provided an analysis over 273 company days and found that the R^2 value of transactions per day regressed against the total volume was 0.85, providing some validity for our result's applicability to the analysis of trading volume.

Comments

While the model provides some parallels to a security trading on the open market, one of its weaknesses is the assumption that uninformed investors cannot infer any information based on the trading behavior and activity of other, possibly-informed investors. For example, in Copeland's model, if the owner of the asset is informed in a negative way, his desire to sell may provide information to uninformed traders that the new news may not be good. As a result, one might not be able to immediately conclude that he can sell the asset to any of the still uninformed investors. The concept of inferring information from the informed is related to the Lemons Principle concept discussed in Section 2.5 which introduces the dynamics of information asymmetry.

Jennings and Barry [19] extended this model by allowing informed traders to take speculative positions, causing prices to adjust more quickly to new information. In this section, we considered the idea that volume is correlated with absolute prices. Another popular belief is that volume is instead correlated with prices rather than the absolute value of prices, meaning that volume is high when prices move upwards, and low when prices move downwards. The following section considers this hypothesis.

■ 3.3 Volume is Heavy in Bull Markets, Light in Bear Markets

Many individuals in finance believe that volume is heavy when the market is going up, and light when it is going down. Karpoff discussed this idea in some detail, citing past works by Epps [10, 11] which showed that the ratio of volume to absolute price change was larger for transactions on upticks than on downticks, both in the stock and bond markets. This trend also held when considered over daily intervals. On the other hand, some researchers found evidence to the contrary. Table 2 summarizies different research on the relationship between price and volume.

Unlike the previous section, past research has been less unanimous on this hypothesis. However, Karpoff later wrote another paper [23] which hypothesized that the short sales constraint contributed to the correlation between volume and price. By analyzing the futures market, which has no constraints on short-selling[16], Karpoff found no significant relationship between trading volume and price changes. This conclusion seems to negate the hypothesis that volume has noticeable correlation with corresponding price changes.

[16] An investor can be on either side of a futures contract with no holding requirement, as the Chicago Board of Trade would not expect an investor shorting a single corn futures contract to actually own 5,000 bushels of corn.

Author(s)	Year of Study	Sample Data	Analysis Period	Interval	Supports Positive Correlation?
Granger Morgenstern	1963	Stock market aggregates 2 common stocks	1939-1961	weekly	no
Godfrey Granger Morgenstern	1964	Stock market aggregates 3 common stocks	1956-1962 1951-1953 1963	weekly daily transactions	no
Ying	1966	Stock market aggregates	1957-1962	daily	yes
Epps	1975	20 NYSE bonds	Jan 1971	transactions	yes
Morgan	1976	17 common stocks 44 common stocks	1962-1965 1926-1968	4 days monthly	yes
Epps	1977	20 common stocks	Jan 1971	transactions daily	yes
Hanna	1978	20 NYSE bonds	May 1971	transactions	yes
Rogalski	1978	10 common stocks 10 associated warrants	1968-1973	monthly	yes
James Edmister	1983	500 common stocks	1975 1977-1979	daily	no
Comiskey Walkling Weeks	1984	211 common stocks	1976-1979	yearly	yes
Harris	1984	50 common stocks	1981-1983	transactions daily	yes
Smirlock Starks	1985	131 common stocks	1981	transactions	yes*
Wood McInish Ord	1985	946 common stocks 1138 common stocks	1971-1972 1982	minutes	no
Harris	1986	479 common stocks	1976-1977	daily	yes
Jain Joh	1986	Stock market aggregates	1979-1983	hourly	yes
Richardson Sefcik Thompson	1987	106 common stocks	1973-1982	weekly	yes

Table 2. Papers which test positive correlation between price change and volume (Karpoff [22]).
*Positive correlation found only on days where there was arrival of information.

4 Serial Correlation of Returns with Abnormal Volume

This section, unlike the previous section which analyzed how price and volume moved together, considers how prices move after periods of unusual volume. We will focus on Campbell et al.'s work [4], and compare it to a different model by Morse [33], who found differing results.

4.1 Price Movements on Private Information

Morse in 1980 determined that periods of abnormally large volume usually had positive autocorrelation of returns. He concluded that his findings were due to the existence of asymmetrical information in the marketplace. In particular, if investors have private information, they may know that the price of a security, which reflects all public information available[17], is incorrect. As a result, these investors will trade heavily on the issue until the price reflects the valuation of the security if the private information became public. Because it may take time for the price to move to this new price, we will observe a price movement in the same direction for potentially a few consecutive days. This monotonic price movement leads to a positive autocorrelation of returns.

To test his hypothesis, Morse took daily price and volume data from 1973 to 1976, inclusive, for 50 securities, 25 from over the counter (OTC), 20 from the NYSE, and 5 from the American Stock Exchange (ASE). Each of these stocks had to have at least 100 shares of trading in 90% of the trading days, and had to have 9 years of earnings data[18]. From this data, the residuals of return and volume were taken from the following equations:

$$R_i = a_i + b_i R_m + r_i$$

and

$$V_i = c_i + d_i V_m + v_i,$$

where R_i, V_i are the daily return and volume of the i^{th} security, R_m, V_m the market return and volume that same day, a_i, b_i, c_i, d_i are regression coefficients, and r_i, v_i are the return and volume residuals. The desire to use residuals is because private information is most likely firm specific; so, removing market-wide non-informational factors will help isolate the effects of the private information.

From the daily returns, arbitrary windows of length L were chosen, with overlapping sequences of T days ($1 < T < L$) considered. From the set of overlapping sequences, the one with the largest sum of residuals which were all strictly positive was chosen as the data point for that L day window[19]. Using these data points of abnormally large volume, tests were made to determine if there was a departure from zero for the serial correlation of price residuals.

Prior research on adjust return residuals found that the probability of a return reversal was approximately 0.5, which asserts that generally the return from one day will not predict the direction of return the following day. Using this assumption, a t-statistic was generated from the normal approximation of the binomial distribution of a random variable taking one when there is a return reversal, and zero otherwise. Table 3 shows the results from Morse's analysis. From the results, it can be seen that the null hypothesis that there is no serial correlation in returns during periods of unusually high volume was rejected for the majority of the sample tests done. From this, we can conclude that for the given dataset, there was likely a serial correlation of returns.

[17] We assume the semi-strong form of the efficient market hypothesis holds.

[18] This last condition was used because Morse used the same securities for another study. However, it did not hurt to do the analysis on more established companies.

[19] Some windows did not have any sequence with T consecutive positive residuals. In those cases, there was no data point for that window.

Sequence Length (T)	Window Length (L)	Samples	Same Sign	Different Sign	Student t-Test
2	50	902	508	394	3.80**
3	50	1382	796	586	5.65**
4	50	1539	866	673	4.92**
5	50	1488	863	625	6.17**
10	50	909	486	423	2.09*
2	100	492	254	238	0.72
3	100	886	514	372	4.77**
4	100	1098	616	482	4.04**
5	100	1128	649	479	5.06**
10	100	774	411	363	1.73
2	200	250	139	111	1.77
3	200	492	278	214	2.88**
4	200	687	386	301	3.24**
5	200	788	452	336	4.13**
10	200	675	372	303	2.66**

Table 3. Tests for serial correlation of returns (Morse [33]).
* Significant at 0.05 level. ** Significant at 0.01 level.

■ 4.2 Mean Reversion from Non-Informational Trading

Campbell et al. [4], on the other hand, considered the returns based on a model of non-informational trading and came up with a different claim. They found that price changes due to high volume tend to be reversed over time. Their hypothesis centers on the idea that non-informational traders sometimes have a need to liquidate assets for external reasons unrelated to their valuation of the holding.

■ 4.2.1 Reasons for Non-Informational Trading

For institutional investors, a number of reasons may cause them to trade. For example, when there is a large outflow of money from shareholders in a mutual fund, money managers may be forced to liquidate assets to satisfy redemptions. If a mutual fund changes its mandate or investment strategy, the fund may have to adjust its holdings accordingly. For index funds, a change in the components of an index would require a portfolio rebalancing. Finally, taxable mutual funds have to consider capital gains concerns at the end of each calendar year. A fund manager may sell securities at losses to cancel the capital gains from transactions earlier in the year, thus avoiding the requirement of passing capital gains distributions to shareholders.

For private investors, they may also make trades near the end of the calendar year for tax reasons. In addition, personal reasons may necessitate the liquidation of assets, such as the purchase of a new home, car, or major appliance, or the need to pay a child's college tuition. All of these reasons can cause supply on a security even if the underlying asset has no change in fundamental valuation.

■ 4.2.2 Risk-Averse Investors as Market Makers

On the other side of the transactions, risk-averse buyers are willing to serve as market makers of sorts, accomodating the sellers' needs provided they receive a higher expected stock return as compensation. This greater return is realized by a drop in price, because the underlying valuation of the security has not changed, yet the stock has been discounted.

Campbell's model assumed that price changes can be classified into two categories, informational and non-informational. In the former case, if information is disseminated to the public, a price change may occur to reflect this news, but little trading will occur if there is a general agreement regarding the new valuation of the security. On the other hand, price changes due to non-informational traders will result in heavier volume, as there will be an imbalance between the perceived valuation of a stock and the value at which it is trading.

Given that this is true, then in the case of non-informational trading, a period of heavy trading will occur where prices will drop (rise) because of the need for liquidation (purchases). However, since the underlying valuation of the issue has not changed, a subsequent reversal of price will occur after the non-informational traders have completed their transactions.

4.2.3 Analysis and Results

Campbell used a value-weighted index of stocks traded on the NYSE and ASE from 7/3/1962 until 12/30/1988[20]. Because of the dominance of the events around 10/19/1987, a day known as Black Monday, where the Dow Jones Industrial Average dropped 508.32 points (22.6%), analysis was truncated to 9/30/1987. Also, with the abolishing of fixed-commission trading on 5/1/1975, modeling turnover would be inaccurate if data before and after that date were combined. Thus, Campbell considered an earlier period, which ended on 12/31/1974, and a latter period, which started on 1/1/1975 and went until 9/30/1987.

The reason that turnover could be markedly different before and after is due to the fact that during days of fixed-commissions, investors might have been less willing to trade as frequently given the high transactions costs. Kador reported that "taking inflation into account, full-service brokerages slashed their rates by fully 95 percent between 1975 and 1998." [21] So, with a factor reduction of 20 in commissions, individuals in the late 90's might trade more frequently than someone in the early 70's.

To accurately measure the market's capacity of absorbing volume, turnover was used instead of raw volume data. To remove low frequency variations in the variance of the data, the log of volume was used. Finally, to detrend the volume data to remove time-varying trends in the amount of turnover, a one-year backward moving average of log turnover was subtracted from our log turnover data, with the result being our volume statistic.

With this data, the first autocorrelation of returns was performed, which showed that a positive autocorrelation existed, and that including which day of the week the data was being taken from further increased the R^2 statistic. In particular, the β found for

$$r_{t+1} = \alpha + \beta r_t$$

was 0.219 for the entire time interval (7/3/1962 - 9/30/1987) with an R^2 of 0.048, and 0.280 with R^2 of 0.079 for 7/3/1962 - 12/31/1974. Adding day of the week components, the R^2 values increased to 0.057 and 0.084 for

$$r_{t+1} = \alpha + (\sum_{d=1}^{5} \beta_d D_d) r_t.$$

The natural positive autocorrelation was surprising, but more interesting was when volume was

[20] They also considered a set of 32 large capitalization stocks and found concurring results.

added to the regression. Performing the following regression

$$r_{t+1} = \alpha + (\sum_{d=1}^{5} \beta_d D_d + \gamma V_t) r_t$$

led to γ values of -0.328 and -0.445 for the entire time interval and the 1962-1974 time intervals, respectively. The corresponding R^2 values were 0.065 and 0.095.

4.2.4 Theoretical Model

Campbell's model assumes an economy with only a risky asset, or stock, and a risk-free asset, or bond. Innovations in the stock price are assumed to be driven by one of three factors:

1. innovation to the current dividend
2. innovation to information about future dividends
3. innovation to the time-varying risk aversion of a subset of investors

The first event causes the stock payoff to be stochastic so that risk-averse investors will require a premium to hold the asset. The second event indicates that prices and dividends alone do not provide the whole story, thus requiring additional information, such as volume. The third event causes changes in the market's overall risk aversion, causing a shift in prices and additional volume caused by increasing risk-averse investors selling their risky stake to other investors. As described at the outset of this subsection, the reallocation of assets is due to non-informational trading, and thus will exhibit a fall in price, followed by a rise in returns.

If we define the economy as one which has an elastic supply of the risk-free asset, then the rate of return will remain constant at r > 0, and define R = 1 + r. Suppose there is a fixed supply of stock, normalized to one share, and that the share pays a dividend at period t of $D_t = \bar{D} + \tilde{D}$, where $\bar{D}$ is the mean dividend, and $\tilde{D}$ is the stochastic portion of the dividend. $\tilde{D}$ follows the following process:

$$\tilde{D}_t = \alpha_D \tilde{D}_{t-1} + u_{D,t},$$

where $0 \leq \alpha_D \leq 1$, and the innovation $u_{D,t}$ is independent, identically distributed (IID), having a zero-mean normal distribution. At time t each investor has information which includes the current price P_t after the dividend, the current dividend D_t, and a signal S_t, which affects during dividend shock in the following manner. The future dividend shock can be written as

$$u_{D,t+1} = S_t + \epsilon_{D,t+1},$$

where S_t and $\epsilon_{D,t+1}$ are jointly IID normal with $E(u_{D,t+1}|S_t) = S_t, \epsilon_{D,t} \sim N(0, \sigma_\epsilon^2), S_t \sim N(0, \sigma_S^2)$.

From this, the present value of the stock can be computed by taking the expected value of the infinite sum of dividends.

$$F_t = E(\sum_{s=0}^{\infty} \frac{D_{t+s}}{R^s} | \tilde{D}_t, S_t) = \frac{R\bar{D}}{r} + \frac{R}{R - \alpha_D} \tilde{D}_t + \frac{1}{R - \alpha_D} S_t.$$

Now, assume that there are two types of investors, type A and type B. Type A investors will never change their risk aversion; so, they have a constant risk aversion parameter a. Type B investors may change over time; so, their risk aversion parameter is written as b_t. During each period, investors want to maximize over the exposure to the risky asset

$$max_{X_t} E_t[-e^{-kW_{t+1}}],$$

where k is the risk aversion parameter, W_t is the wealth at time t, X_t is the holding of the risky asset at time t, and W_{t+1} is

$$W_{t+1} = W_t R + X_t(P_{t+1} + D_{t+1} - RP_t).$$

If we define Z_t as the risk aversion of the marginal investor,

$$Z_t \equiv \frac{ab_t}{(1-p)a + pb_t},$$

where p is the fraction of type A investors. Letting

$$Z_t = \bar{Z} + \tilde{Z}_t$$

and assuming that $\tilde{Z}_t$ follows an AR(1) process; i.e.

$$\tilde{Z}_t = \alpha_Z \tilde{Z}_{t-1} + u_{Z,t},$$

with $0 \leq \alpha_Z \leq 1$, and innovation $u_{Z,t}$ independent of the other shocks, IID, and normal, Campbell proves that there exists an equilibrium price for the stock which is of the form

$$P_t = F_t - D_t + (c_0 + c_Z Z_t),$$

where c_0, c_Z are less than 0.

Implications
When Z_t is high, type B investors are highly risk averse and not willing to hold much stock. As a result, the price will be reduced to induce type A investors to hold a larger quantity of stock. Practically, Z_t and S_t are not directly observable. However, a reduction in price is either caused by an increase in Z_t or or a lower realization of S_t, which would cause a reduction in F_t. Changes due to S_t will not cause much trading, as a reduction in future cash flows will lead everyone to adjust the expected value of the stock correspondingly, resulting in little trading activity. Changes due to Z_t will generate trading among investors. With the decrease in price, the expected return between periods t and t+1 will be higher, leading to a reversal in return of the stock.

4.2.5 Analysis

A valid question at this point might be, *Why do these two papers give different results, and more importantly, which should I believe?* The answer is not obvious or simple, but we will proceed by discussing why the results were different, and then in Section 6, we will perform a test on current market data to see if we can reach either conclusion.

First of all, the analyses were performed over different time frames and different securities. Morse took a four year window (1973-1976) and analyzed 50 stocks that had a nine-year history of earnings, while Campbell took a larger window of time (1962-1987) and also broke it down over 1962-1974 and 1975-1987, looking at a value-weighted index and also 32 large cap stocks.

Another factor which likely could have impacted results was the removal of fixed commissions on May 1^{st}, 1975. Morse's time window is almost split in half by this event, while Campbell's full time-window also crosses this event. Data obtained from the NYSE's website [18] shows that the percentage turnover on the exchange in 1974 was 16%, with the maximum turnover from 1960-1974 being 24% in 1968, while the turnover in the first full year after the removal of fixed commissions saw a turnover of 23%, and since 1980, we have not seen turnover under 30%, with turnover increasing up to 94% in 2001. Campbell does provide an adjustment to their turnover data while Morse does not.

Regarding Campbell's negative regression β's, it should be noted that the R^2 values for each of the results we list were less then 0.10, making the trend not necessarily significant. And in both cases, it is not clear how accurate their models are with regards to the time it takes for private information to manifest itself in the price, in the case of Morse, and the time required for the prices to revert after non-informational selling, in the case of Campbell.

5 Inferring the Hedging Portfolio from Prices and Volume

Karpoff's work looked at price and volume moving together, while Campbell's work considered the predictive nature of volume on future returns of securities. In the work we consider in this section, Lo and Wang [28] discuss using volume data to infer and construct a hedging portfolio, and then assert that this portfolio can predict the future returns of the market portfolio.

Unlike the assumptions of CAPM, which assert that everyone holds only the market portfolio, Lo considers the intertemporal (multi-period) CAPM model which requires that investors hold a hedging portfolio to adjust their exposure to changes in market conditions that affect future returns. These changes affect the investors beyond the single period that investors under CAPM are only interested in.

5.1 Definitions and the Economy

We start with an economy having a discrete set of dates, indicated by t. There are J risky assets in the economy which pay dividends (D_{jt} is the dividend that stock j pays at time t), and one risk-free asset, which has an interest rate of r, which we will assume to be constant and independent of supply and demand. Normalize the number of shares of each stock to 1, and if we denote the stock holdings of an investor as S = $[S_1, S_2, ..., S_J]$, then the market portfolio, S_M, is defined to be $S_M = [1, 1, ..., 1] = \iota$. Similarly for price and dividends, P_t, D_t will represent J X 1 vectors of prices and dividends, respectively. Suppose there are I investors in the economy and at the beginning are given equal shares of stocks and no risk-free holdings. At each period, each investor i seeks to maximize his expected utility, defined by

$$E_t[-e^{W^i_{t+1} - (\lambda_X X_t + \lambda_Y Y^i_t) D_{M(t+1)} - \lambda_Z (1+Z^i_t) X_{t+1}}]$$

where W^i_{t+1}, the investor's wealth in the next period, is

$$W^i_{t+1} = (1+r)W^i_t + (S^i_t)^T [D_{t+1} + P_{t+1} - (1+r)P_t]$$

$\lambda_X, \lambda_Y, \lambda_Z$ are non-negative constants, and X_t, Y^i_t, Z^i_t are one-dimensional state variables. The clearance of the market requires the constraint

$$\sum_{i=1}^{I} S^i_t = \iota$$

We also assume that

$$\sum_{i=1}^{I} Y^i_t = 0$$

along with

$$\sum_{i=1}^{I} Z^i_t = 0$$

The external shocks, D_t, X_t, and Y^i_t, Z^i_t for all i, are assumed to be zero mean independent, identically distributed random variables. We also assume that D_t and X_t are jointly normally distributed with the covariance of D_t positive definite.

The model is such that the investor's utility function does not depend only on his wealth, but also on the dividend payoffs of the stocks. Lo describes this as a "market spirit" and can be best described as investor's reaction to how the stock market is doing as a whole. This spirit is measured by $\lambda_X X_t + \lambda_Y Y_t^i$. When it is positive, investor i extracts positive utility, or pleasure, when the market is doing very well. In effect, the individual might feel that a strong market bodes well for his job security and thus is happy even though he does not directly profit from the market's large gains. Because of this inherent increase in utility, this investor will be less likely to increase his holdings in the stock market. X_t is market-wide, and indicates the overall sentiment of all investors. As a result, this will affect the equilibrium prices of the market. In particular, in the economy defined, prices will have a linear equilibrium of the form[21]

$$P_t = -a - bX_t$$

where b is

$$b = \lambda_X[(1+r) + \lambda_Z \sigma_{XD}\iota]^{-1}\sigma_{DD}\iota$$

and if we introduce the excess dollar return on the stocks to be

$$Q_{t+1} \equiv D_{t+1} + P_{t+1} - (1+r)P_t$$

and return rate $R_{j(t+1)} \equiv \frac{Q_{j(t+1)}}{P_{jt}}$, a is

$$a = \frac{1}{r}(\frac{1}{I}\sigma_{QQ}\iota + \lambda_Z \sigma_{QX})$$

where

$$\sigma_{QQ} = \sigma_{DD} - (b\sigma_{XD} + \sigma_{DX}b') + \sigma_X^2 bb'$$

and

$$\sigma_{QX} = \sigma_{DX} - \sigma_X^2 b$$

Thus, X_t directly affects the market price of securities, as it affects everyone's utility function. On the other hand, the Y_t^is are considered to be idiosyncratic differences among investors in their feelings about the market. Since it has an aggregate sum of zero, it does not affect prices, but it does affect how individual investors will adjust their holdings.

Using our model, the dollar return vector can be expressed as

$$Q_{t+1} = ra + (1+r)bX_t + \tilde{Q}_{t+1}$$

where

$$\tilde{Q}_{t+1} \equiv D_{t+1} - bX_{t+1}$$

is the vector of unexpected dollar returns on the stocks. From this, it can also be shown that

$$S_H \equiv \sigma_{QQ}^{-1}\sigma_{QX}$$

■ 5.2 Two-factor Turnover Structure

One of the propositions proven in the first of Lo's trading volume papers [27] was that when trading in the hedging portfolio is small relative to that in the market portfolio, two-fund separation in the holdings leads to an approximate two-factor structure in turnover; that is,

$$\tau_t \approx \iota F_{Mt} + S_H F_{Ht}$$

[21] Proof in Lo [28].

where S_H is the holding in the hedging portfolio, and F_{Mt}, F_{Ht} represent the turnover in the market and hedging portfolios, respectively. So, if we can empirically determine F_{Mt} and F_{Ht}, then the loadings on the second term will allow us to identify the composition of the hedging portfolio.

Furthermore, for an arbitrary portfolio S, its dollar return at time period t, denoted $Q_{S_t} \equiv S'Q_t$, can be used to predict dollar return of the market for the following period.

$$Q_{M(t+1)} = \delta_0 + \delta_1 Q_{S_t} + \epsilon_{M(t+1)}$$

As discussed in Appendix A.6.2, the coefficient of determination, R^2, measures the predictive power of this portfolio for future market dollar returns.

Theorem 1. *The dollar return of the hedging portfolio, S_H, provides the best forecast for the future dollar return of the market.*

Proof. Proof in Appendix B.1. ■

The following section shows the empirical results that test the validity of this theorem.

5.3 Empirical Results

Weekly return and turnover data for individual stocks were taken from July 1962 to December 1996 and separated into seven five-year subperiods in an effort to manage the nonstationary of aggregate turnover. To identify the two common factors F_{Mt} and F_{Ht} described in the previous section, an approximate two-factor model was first created

$$\tau_{jt} = F_{Mt} + \theta_{Hj} F_{Ht} + \epsilon_{jt}$$

for j = 1, 2, ..., J, with θ_{Hj} the percentage of shares of stock j held in the hedging portfolio and ϵ_{jt} the error term, assumed to be independent across all j.

5.3.1 Estimating the Hedging Portfolio

To identify the hedging portfolio, we used the equal-weighted turnover and share-weighted turnover indexes as representatives. Defining N_j as the number of shares outstanding for stock j and N as the sum of all the shares outstanding, the equal-weighted turnover index was defined as

$$\tau_t^{EW} \equiv \frac{1}{J} \sum_{j=1}^{J} \tau_{jt} = F_{Mt} + n^{EW} F_{Ht} + \epsilon_t^{EW},$$

where $n^{EW} = \frac{1}{J} \sum_{j=1}^{J} \theta_{Hj}$ and ϵ_t^{EW}, the error term. Similarly, the share-weighted turnover was defined as

$$\tau_t^{SW} \equiv \sum_{j=1}^{J} \frac{N_j}{N} \tau_{jt} = F_{Mt} + n^{SW} F_{Ht} + \epsilon_t^{SW},$$

with $n^{SW} = \sum_{j=1}^{J} \frac{N_j}{N} \theta_{Hj}$ and ϵ_t^{SW}, the error term. Because of the independence assumption for the ϵ's, these terms become small as J increases. These two equations yield

$$\tau_{jt} = \beta_{\tau j}^{SW} \tau_t^{SW} + \beta_{\tau j}^{EW} \tau_t^{EW} + \epsilon_{jt},$$

where

$$\beta_{\tau j}^{EW} = \frac{n^{EW} - \theta_{Hj}}{n^{EW} - n^{SW}}$$

and

$$\beta_{\tau j}^{SW} = \frac{\theta_{Hj} - n^{SW}}{n^{EW} - n^{SW}}.$$

The equations above have the inherent constraints that $\beta_{\tau j}^{EW} + \beta_{\tau j}^{SW} = 1$ for all j, and $\sum_{j=1}^{J} \beta_{\tau j}^{EW} = 0$. Using the equation for τ_{jt} and these constraints, a constrained regression could be performed on the data, and from the estimates of $\beta_{\tau j}^{EW}$, denoted $\hat{\beta}_{\tau j}^{EW}$, we can create the portfolio weights of the hedging portfolio according to $\hat{\theta}_{Hj} = (n^{EW} - n^{SW})\hat{\beta}_{\tau j}^{EW} + n^{SW}$. Since the hedging portfolio has two free parameters[22], we can let $n^{SW} = 1$ and $n^{EW} - n^{SW} = \phi$, where ϕ is a parameter we can calibrate, representing the degree of deviation from the market portfolio. Thus, from the volume data, we can estimate the hedging portfolio.

5.3.2 Forecasting Market Returns

After creating the hedging portfolio, the next step was to see how well it predicted future market returns and also the cross-section of expected returns in comparison with other factors. Empirically, it was shown that the future market returns were best predicted by the hedging portfolio. As for the expected returns of portfolios sorted by deciles, the following analysis was done. First, following the well-regarded regression tests discussed by Fama and MacBeth [13], portfolios sorted by an estimated parameter were formed in the first time period, β's were estimated for these portfolios in the second time period, and a cross-sectional regression was run in the third time period. The time periods were the selected five-year intervals from the original data set. Using only securities which existed over all three time periods, a bivariate regression was run for

$$R_{jt} = \alpha_j + \beta_j^M R_{Mt} + \beta_j^H R_{Ht} + \epsilon_{it}$$

In the first period, 100 portfolios were created corresponding to the deciles of the estimated market and hedging-portfolio betas. The two betas were estimated for each of the 100 portfolios in the second period, and the following cross-sectional regression was estimated in the third period:

$$R_{pt} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_p^M + \gamma_{2t}\hat{\beta}_p^H + \eta_{pt}.$$

where p = 1, 2, ... , 100 (portfolio number) and R_{pt} was the equal-weighted portfolio return. The results from using hedging portfolio return beta and dollar return beta are shown in Table[23]4.

From the results, it can be seen in the 1977-1981 period that the coefficients to the return β's are statistically significant, with t-statistics of -3.712 and -4.140 for return and dollar return, respectively. Comparatively, the market β's coefficients had t-statistics less than one in magnitude. Using regressions with two other popular β's, namely the small-minus-big cap portfolio return ($\hat{\beta}_p^{SMB}$) and the optimal forecast portfolio ($\hat{\beta}_p^{OFP}$), Lo found that the t-statistic for SMB was 4.433, and thus significant, while that for OFP was 0.632. Of the other periods, none of the hedging portfolio coefficients to β were significant, while only the 1992-1996 period coefficients for SMB (t-statistic = 2.147) and OFP (2.407) were significant. Lo concludes by saying "the point estimates of the cross-sectional regressions show that the hedging-portfolio factor is comparable in magnitude and in performance to other commonly proposed factors" for return predictability.

5.3.3 Comments

The approach of using volume data to determine the hedging portfolio in this work was novel. The economy model was reasonably general, except for the fact that investors maximized their wealth

[22] Assuming these parameters are not observed, we have freedom to choose them.
[23] Table taken from Lo [28].

Model	Years	Statistic	γ_{0t}	γ_{1t}	γ_{2t}	$\bar{R}^2$
Return	1/72 - 12/76	Mean:	0.004	-0.002	-0.002	14.3
($\phi = 1.25$)		Std:	0.035	0.035	0.037	10.9
		t-Stat:	2.040	-1.047	-0.820	
Dollar	1/72 - 12/76	Mean:	0.004	-0.002	-0.104	15.5
Return		Std:	0.032	0.034	3.797	10.9
($\phi = 1.50$)		t-Stat:	2.162	-1.081	-0.442	
Return	1/77 - 12/81	Mean:	0.003	-0.001	-0.012	13.1
($\phi = 4.75$)		Std:	0.014	0.020	0.051	12.4
		t-Stat:	3.748	-0.902	-3.712	
Dollar	1/77 - 12/81	Mean:	0.003	-0.001	-1.564	12.5
Return		Std:	0.013	0.020	6.104	12.2
($\phi = 4.25$)		t-Stat:	3.910	-0.754	-4.140	
Return	1/82 - 12/86	Mean:	0.006	-0.001	-0.006	9.6
($\phi = 1.75$)		Std:	0.011	0.020	0.055	9.4
		t-Stat:	8.390	-0.780	-1.732	
Dollar	1/82 - 12/86	Mean:	0.006	-0.002	-0.740	10.4
Return		Std:	0.011	0.019	19.874	9.5
($\phi = 2.00$)		t-Stat:	8.360	-1.297	-0.602	
Return	1/87 - 12/91	Mean:	0.002	0.000	0.000	5.4
($\phi = 47$)		Std:	0.016	0.019	0.060	6.1
		t-Stat:	2.254	-0.147	0.168	
Dollar	1/87 - 12/91	Mean:	0.002	0.000	0.189	6.0
Return		Std:	0.016	0.019	18.194	6.7
($\phi = 20$)		t-Stat:	2.434	-0.147	0.168	
Return	1/92 - 12/96	Mean:	0.002	0.001	-0.004	6.9
($\phi = 38$)		Std:	0.013	0.020	0.091	6.8
		t-Stat:	2.785	1.164	-0.650	
Dollar	1/92 - 12/96	Mean:	0.003	0.000	-1.584	6.2
Return		Std:	0.015	0.022	12.992	6.6
($\phi = 27$)		t-Stat:	3.279	-0.178	-1.970	

Table 4. Cross-sectional regression tests of market and hedging portfolio β's for five-year subperiods. $\bar{R}^2$ is average R^2 over the 100 portfolios.

in a myopic (next period only) time frame. However, having a utility function with dependence on future time periods would make the analysis much more difficult. The stock market data was separated to reduce the effects of non-stationary in turnover data. As discussed in the previous section, turnover on the NYSE has increased dramatically over the decades. However, as with Morse's analysis, one of the time periods crossed the date when commissions were abolished (May 1^{st}, 1975), causing that group of data to possibly be non-stationary. As for using the hedging portfolio to predict future returns, the empirical data seemed to indicate that neither this method, nor any of the other β's considered (SMB and OFP), could adequately predict the future cross-section of expected returns.

6 An Analysis of Current Data

In papers by Campbell and Morse, the question was if there was some serial correlation of returns around instances of large volume. The two authors came up with differing results, and in Section 4.2.5 we discussed possible reasons why the results were different. One of the main issues was that turnover data is likely non-stationary due to many market factors, regulatory rules changes, and potentially changes in tax law. With that in mind, we thought we would perform a brief analysis of current market data to see what the current conditions seem to be[24].

In Section 3, we looked at arrival of new information and considered what happens when there is a disagreement about the information presented. A recent study by Meschke [32] found that stocks typically run up 2.4% in the two days before a CEO appears on CNBC, and 1.9% on the day after. But the stock loses 3.26% in the 10 days after that. Yet, CEO interviews on CNBC are considered to be "non-events". Meschke concluded that "enthusiastic investors create transitory buying pressure" much the same way non-informational traders create selling pressure when they have to liquidate.

In addition, past work has considered the change in volume and price around quarterly earnings announcements. For example, Lobo and Tung [29] found that trading volume reaction to earnings announcements was related to the amount of information asymmetry before the event, and the subsequent price movement. Oppenheimer and Grissom [35] analyzed a similar phenomenon with real estate investment trusts (REITs).

Since there is some general agreement that earnings announcements are generally informative, and since it can be seen from a cursory glance that trading volume tends to be high near such announcements, we decided to see if any type of serial correlation existed today between volume reaction and subsequent price changes around an earnings announcement.

6.1 Testing the Hypotheses

Campbell's paper [4] stated that "price changes accompanied by high volume will tend to be reversed." On the other hand, Morse concluded exactly the opposite due to his theory of the permeation of private information into the stock price. From our cursory analysis around earnings announcements, we will see that Campbell's explanation appears more likely in our current market.

6.2 Dataset Used

To test this hypothesis, a set of data had to be selected. We chose the thirty stocks in the Dow Jones Industrial Average (DJIA) as a representative sample of equities, and found the date of each of their third quarter 2002 earnings announcements, which were released between October 4^{th}, 2002 and November 19^{th}, 2002. A list of these thirty stocks and their earnings announcement date may be found in Table 5.

6.3 Analysis

Since earnings announcements can happen before the market opens or after it closes[25], we count the number of trading days before (after) the earnings released based on the days before (after) the information was released. So, an earnings announcement after hours on January 21^{st}, 2003

[24]This research of our own is in no way comprehensive. Rather, it is a small test to get some idea about market behavior and a sample of the type of work we can do to better understand our current market.

[25]Earnings announcements can happen during trading hours, too, but most firms today prefer to present new information while markets are closed so investors can analyze and digest it properly before rushing to adjust their positions in the open market.

Company	Ticker	Earnings Date	Before/After Trading Day
3M Co.	MMM	10/21	Before
Alcoa Inc.	AA	10/4	Before
American Express Co.	AXP	10/28	After
AT&T Corp.	T	10/22	Before
Boeing Co.	BA	10/16	Before
Caterpillar Inc.	CAT	10/16	Before
Citigroup Inc.	C	10/15	Before
Coca-Cola Co.	KO	10/16	Before
E.I. DuPont de Nemours & Co.	DD	10/21	Before
Eastman Kodak Co.	EK	10/24	Before
Exxon Mobil Corp.	XOM	10/31	Before
General Electric Co.	GE	10/11	Before
General Motors Corp.	GM	10/15	Before
Hewlett-Packard Co.	HPQ	11/20	After
Home Depot Inc.	HD	11/19	Before
Honeywell International Inc.	HON	10/16	Before
Intel Corp.	INTC	10/15	After
International Business Machines Corp.	IBM	10/16	After
International Paper Co.	IP	10/23	Before
J.P. Morgan Chase & Co.	JPM	10/16	Before
Johnson & Johnson	JNJ	10/15	Before
McDonald's Corp.	MCD	10/22	Before
Merck & Co. Inc.	MRK	10/18	Before
Microsoft Corp.	MSFT	10/17	After
Philip Morris Cos. Inc.	MO	10/17	Before
Procter & Gamble Co.	PG	10/29	Before
SBC Communications Inc.	SBC	10/24	Before
Wal-Mart Stores Inc.	WMT	11/13	Before
Walt Disney Co.	DIS	11/7	After

Table 5. Components of the Dow Jones Industrial Average (Co = Company, Corp = Corporation, Inc = Incorporated).

and one before the market opened on January 22^{nd}, 2003 would both have their first trading day after information release on January 22^{nd}. Also, since earnings announcements can come any day of the week, we analyze returns for a five trading day window before and after the earnings announcement so that all of the analyses will include a weekend in the five day intervals. This removes the possibility that a stock with an earnings announcement early or late in the week will have a weekend inside their window of analysis while another security with a Wednesday release will not include a weekend. Also, third quarter earnings were chosen since none of these windows included any holidays where the market either was not open, or only traded for half of a day. Finally, to compare the excess return from these securities, the returns in these five-day windows were adjusted by the return of the Standard and Poors 500 index to remove any performance due to macroeconomic events, or market movements as a whole. Also, the returns were adjusted to reflect dividends.

The volume data obtained was the average number of shares traded in the five trading days before an earnings announcement. This was compared with the average daily trading volume in 2002, according to data obtained from Yahoo! Finance[26] [15]. Table 6 shows the volume data along with return information around the five-day windows of the earnings release.

■ 6.4 Results

From the data, it can be seen that twenty-six of the thirty securities saw an increase in trading volume in the five days preceding an earnings announcement, with the average increase in trading during that period was 24.93%. Six had volume greater than 50% of the average daily volume.

The average excess return in the five days preceding an earnings announcement was negligible, at -0.05% (standard deviation 5.93%), while the excess returns after earnings announcements averaged 1.32% (standard deviation 6.70%), not significantly above the market return. Looking at the excess returns before and after the earnings announcement, we see that fourteen of the thirty stocks saw a sign change, while sixteen did not, about half of what one would expect.

While no significant trends can be concluded, it can be seen that the three securities that had the largest increase in trading volume, Alcoa, General Electric, and General Motors, all had sign changes and greater than 10% return differences between the time period after the earnings announcement and the period before earnings. This tends to support Campbell's hypothesis that of the stocks which saw heavy trading before their earnings announcement, a negative autocorrelation of returns can be seen. This would indicate that the pre-earnings trading was likely non-informational, and that the quantity of asymmetrical information was small.

The argument for this conclusion is that information today is more readily available than in the past, reducing the potential amount of asymmetry in the market. Further, companies recently tend to preannounce any earnings adjustments well before the actual earnings announcement release, reducing the amount of new information earnings announcements may provide. For example, Philip Morris actually had a 6% decline in volume in the five days before its 3rd quarter earnings announcement, but had volume four and a half times normal on November 12^{th}, 2002, when it announced a warning for reduced future earnings.

Granted, these results come from a cursory analysis of the relationship between returns and volume around a particular information release from the most recent quarter of earnings data. To reach adequate conclusions, a larger set of securities should be considered, and a larger set of informational events need to be analyzed. The choice of earnings announcements was a convenient event which was guaranteed to occur for each security once every three months.

[26] This was the case for all stocks except for AT&T, who had a 1 for 5 reverse stock split on 11/19/2002. For this security, the average daily volume from 1/1/2002 until 10/31/2002 was used instead.

Ticker Symbol	2002 Daily Volume	Avg Volume Pre-Earnings	Percent Increase	Adj % Return Pre-Earnings	Adj % Return Post-Earnings
AA	3,577,363	6,269,600	75.26	4.92	-5.48
GE	25,958,863	44,098,600	69.88	-6.49	8.79
JNJ	7,527,500	10,588,500	40.66	-4.81	-2.64
MSFT	39,818,136	59,645,900	49.80	0.09	0.53
AXP	4,847,818	5,949,440	22.72	0.51	7.39
GM	6,151,954	11,268,600	83.17	-13.49	4.04
JPM	12,439,181	19,959,800	60.46	1.59	5.92
PG	3,948,090	4,652,800	17.85	-5.99	-1.14
BA	3,727,727	5,006,900	34.32	-8.99	-7.19
HD	14,386,272	11,156,700	-22.45	6.07	-14.12
KO	5,820,227	5,844,900	0.42	-8.76	-9.68
SBC	8,023,636	10,647,000	32.70	6.70	1.04
C	17,786,681	23,502,100	32.13	5.83	9.60
HON	4,235,181	5,408,700	27.71	2.60	3.52
MCD	7,362,045	6,339,000	-13.90	-4.76	1.18
T	16,797,000	12,283,000	-26.87	-1.77	8.49
CAT	1,878,090	2,963,500	57.79	-5.24	8.91
HPQ	12,530,818	13,445,000	7.30	0.98	13.20
MMM	2,345,909	2,892,780	23.31	-1.86	0.43
UTX	2,458,272	3,152,600	28.24	0.31	8.57
DD	2,878,454	3,009,500	4.55	-1.59	-0.64
IBM	9,361,363	13,790,000	47.31	6.44	10.31
MO	9,561,409	9,021,400	-5.65	-4.82	6.61
WMT	9,395,500	7,736,500	-17.66	2.59	-3.24
DIS	7,772,818	9,221,400	18.64	7.30	-0.94
INTC	63,161,909	80,077,000	26.78	13.23	-9.39
MRK	7,061,136	7,402,600	4.84	0.86	-0.88
XOM	11,920,500	12,010,000	0.75	-2.40	-0.66
EK	2,439,409	3,855,800	58.06	7.83	2.01
IP	2,614,500	2,867,000	9.66	1.67	-4.95

Table 6. Average daily volume, 5-day pre-earnings average volume, percentage increase of volume near earnings, and adjusted returns over five trading days, before and after the earnings announcement for DJIA stocks.

7 Conclusions

Trading volume has been shown to have a relationship to securities prices in a few different ways. Karpoff and others modeled and determined empirically that large turnover occurred during times of large absolute price change, although significance could not be found for the direction of this price change. Campbell and Morse considered how abnormal volume would affect the autocorrelation of returns. Campbell used a model of non-informational traders to explain the return reversals he found in empirical data. Morse, on the other hand, found positive autocorrelations and explained that this behavior in prices could happen if a subset of individuals on the market had private information and traded on it until the price reflected this information. The fact that these two authors had differing results could stem from a variety of reasons, but perhaps primarily by the time frame of their analyses, as regulatory changes and availability of information have changed over time. Finally, Lo asserted that two-fund separation in portfolio holdings would result in an approximate two factor structure for turnover, and from this information, one could determine the hedging portfolio. Furthermore, this portfolio was the best predictor of future market returns, both theoretically and empirically, although Lo showed that the hedging portfolio was only as good as other possible portfolios in their predictive power of the cross-section of expected returns.

A Statistics Review and Overview

The following section provides a brief overview of transform theory and different statistical techniques [31] which are referenced in the trading volume research. Hypothesis tests and regressions are two very common statistical techniques employed by researchers. Within the realm of hypothesis testing, statisticians can use p-values, t-statistics, Chi-square tests, and F-tests to analyze their data. A reader with an understanding of these topics need not read this section.

A.1 Transforms of Random Variables

Sometimes analyses of some combination of random variables can be unwieldy. For instance, finding the probability density function (PDF) of the sum of two random variables involves a convolution of the individual PDFs. For discrete probability mass functions (PMF), a convolution is also required. This computation can be simplified if we introduce the exponential transform, also known as the s-transform, in continuous time, and the z-transform in discrete time.

A.1.1 Definitions

The s-transform of a PDF $f_x(x)$ is defined as

$$f_x^T(s) = E(e^{-sx}) = \int_{-\infty}^{\infty} e^{-sx_o} f_x(x_o)dx_o$$

while the z-transform of a PMF $p_x(x)$ is

$$p_x^T(z) = E(z^x) = \sum_{k=-\infty}^{\infty} z^k p_x(k)$$

A.1.2 Convolution

One of the useful properties of these transforms is that it converts a convolution sum/integral into a product. If x and y are two continuous independent random variables with PDFs $f_x(x)$ and $f_y(y)$, and we wish to determine the PDF of their sum z = x + y, we know from Fourier series that $f_z(z) = f_x(z) * f_y(z)$, where the operator is a convolution. However, if we do the mathematics in tranform space, we obtain

$$f_z^T(s) = E(e^{-sz}) = E(e^{-s(x+y)}) = E(e^{-sx}e^{-sy}) = \int_{x=-\infty}^{\infty}\int_{y=-\infty}^{\infty} e^{-sx}e^{-sy} f_{x,y}(x,y)dxdy$$

since x and y are independent, $f_{x,y}(x,y) = f_x(x)f_y(y)$; so, we have

$$f_z^T(s) = \int_{x=-\infty}^{\infty} e^{-sx} f_x(x) \int_{y=-\infty}^{\infty} e^{-sy} f_y(y) = f_x^T(s) f_y^T(s)$$

In discrete time, the integrals are converted to sums, and we once again have

$$p_z^T(z) = p_x^T(z) p_y^T(z)$$

A.1.3 Moment-Generating Properties

Another useful properties of tranforms are their moment-generating characteristics. Here we will first show the steps necessary for discrete time, with the continuous time case following below. To compute the moments of a discrete random variable x, we can do so by taking derivatives of the PMF.

$$\frac{d}{dz}p_x^T(z) = \frac{d}{dz}E(z^x) = E(xz^{x-1})$$

When this is evaluated at z = 1, we have E(x). If we take another derivative,

$$\frac{d^2}{dz^2}p_x^T(z) = E(x(x-1)z^{x-2})$$

which, when evaluated at z = 1, yields $E(x^2) - E(x)$. So, the variance can be determined by the E(x(x-1)) + E(x) - $E(x)^2$.

In continuous time, the general equation is

$$\frac{d^k}{ds^k}f_x^T(s) = \frac{d^k}{ds^k}E(e^{-sx}) = (-1)^k E(s^k e^{-sx})$$

When evaluated at s = 0, we have the k^{th} derivative yielding $\pm E(x^k)$.

■ A.2 Hypothesis Testing

The idea of hypothesis testing involves first developing a conjecture, or hypothesis. This is generally termed the *null hypothesis*. With any hypothesis, there exists the possibility that it is false. Any such contradiction of the null hypothesis is called the alternative hypothesis. Now, to determine which of these possibilities is true, we must have some information. The information we use to evaluate our hypothesis is called the *test statistic*. Different test statistics exist depending on the hypothesis and the type of data we have available. In almost all cases, it is useful to have an ensemble of measurements that we use to obtain our test statistic. The reason for this comes from the result of the Central Limit Theorem.

■ A.2.1 Central Limit Theorem

The Central Limit Theorem states that the average of a large number of independent samples from a distribution will itself approach a normal distribution. Mathematically, we write:

Theorem 2. *If $\bar{X}$ is the mean of a random (independent) sample $X_1, X_2, X_3, ..., X_n$ of size n from a distribution with a finite mean μ and a finite positive variance σ^2, then the distribution of*

$$W = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{\sum_{i=1}^{n} X_i - n\mu}{\sqrt{n}\sigma}$$

is normal with zero mean and unit variance as $n \to \infty$ [17].

Proof. [9] Let y = $\sum_{i=1}^{n} X_i$. Consider z = $\frac{y-E(y)}{\sigma_y}$, where E(y) = nμ and $\sigma_y = \sqrt{n}\sigma$, the latter because the samples are independent. Apply the s-transform to z to obtain

$$f_z^T(s) = E(e^{-sz}) = E(e^{-\frac{sy}{\sigma_y} + \frac{sE(y)}{\sigma_y}}) = E(e^{-\frac{sy}{\sqrt{n}\sigma} + \frac{sn\mu}{\sqrt{n}\sigma}}) = f_y^T(\frac{s}{\sqrt{n}\sigma})e^{\frac{s\sqrt{n}\mu}{\sigma}} = (e^{\frac{s\mu}{\sqrt{n}\sigma}} f_x^T(\frac{s}{\sqrt{n}\sigma}))^n$$

with the last equality resulting from the identity $f_y^T(s) = [f_x^T(s)]^n$ for y = $\sum_{i=1}^{n} X_i$ when the X_i's are independent. Now, applying Taylor series approximations to the two terms, we obtain[27]

$$e^{\frac{s\mu}{\sqrt{n}\sigma}} \approx 1 + \frac{\mu s}{\sigma\sqrt{n}} + \frac{\mu^2 s^2}{2\sigma^2 n} + O(s^3)$$

[27] O(s^k) means additional terms of order k or greater.

and

$$f_x^T(\frac{s}{\sqrt{n}\sigma}) \approx 1 - \frac{\mu s}{\sigma\sqrt{n}} + \frac{E(X^2)s^2}{2\sigma^2 n} + O(s^3)$$

so their product, for small s, yields

$$f_z^T(s) \approx (1 + \frac{s^2/2}{n})^n$$

which, as n $\rightarrow \infty$, converges to $e^{s^2/2}$, which is the s-transform for a zero-mean, unit normal Gaussian. ■

Again, the Central Limit Theorem asserts that a large number of samples from a given distribution will have an average that possesses a Gaussian-like distribution. The advantage of having a normal distribution is that the probability density function of this distribution is well-known, and thus we may gauge the probability that a particular average can occur given certain assumptions of the first two moments (mean and variance) of the underlying distribution.

So, from a set of measurements and a given hypothesis, four possible situations can occur. If the null hypothesis H_0 is true, from the test statistic we can conclude correctly that H_0 is true, or else we can erroneously reject H_0. The latter is classified a Type I error (having probability α of occurring). On the other hand, if H_0 is false, we can either correctly reject H_0, or erroneously accept it. Such acceptance is classified as a Type II error (with probability β).

■ A.2.2 p-Value

To determine whether or not to accept a hypothesis, we must ask, *"What is the maximum value of α we are willing to tolerate and yet still believe that H_0 is true?"* Often, we choose p = 0.025, which means that we will accept H_0 if our test statistic yields a Type I error with less than probability p. If the test statistic does not fall within the rejection region, then we say that the null hypothesis is true with confidence 100(1-p)%.

■ A.2.3 Law of Large Numbers

Related to the Central Limit Theorem is the law of large numbers. In its weakest form, the law of large numbers states that the average of a sequence of random variables having (the same) finite expected value will converge to this expected value. An example of the application of this law may be found in Section 2.2.

■ A.3 t-Statistics

When dealing with samples of data, the variance of the data could be unknown. As a result, the sample variance is computed and used as an estimate of the variance. Mathematically,

$$z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

where z is what we ideally want to analyze. In reality, we have

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

where s is the sample standard deviation. The variability of t depends on how many samples we have (to see this, consider the degenerate case of trying to calculate the sample variance with only one sample). This dependence is called the *degrees of freedom* and is equal to (n-1), where n is the number of samples. The t-distribution approaches the normal distribution as n $\rightarrow \infty$, but is always less probable around the mean and more probable in the tails for finite n. So, while we have $z_{\alpha=0.025} = 1.96$, in the case where we have 5 samples, $t_{\alpha=0.025} = 2.776$, which means that the region where α is 0.025 is the region greater than or equal to 2.776 standard deviations greater than the mean.

In our analysis, we will see that two particular levels of significance used are 0.05 and 0.01. A t-statistic greater than 1.96 has a level of significance greater than 0.05. One which is greater than 2.576 has a level of significance greater than 0.01.

A.4 Chi-square Tests

The goal of a chi-square test is to measure the degree of disagreement between the observed measurements and the null hypothesis. The chi-square test seeks to analyze if the variability, i.e. sample variance, is consistent to the hypothesized value. The quantity $\frac{(n-1)s^2}{\sigma^2}$ is known to have a chi-square χ^2 distribution if the sampled population is normally distributed. To understand the chi-square distribution, we must first discuss the gamma function and the gamma distribution. The gamma function is defined as

$$\Gamma(t) = \int_{y=0}^{\infty} y^{t-1} e^{-y} dy$$

for t > 0. Note, $\Gamma(1) = 1$ and $\Gamma(n) = (n-1)!$ for integer n. Now, we define the gamma distribution to be the probability density function (pdf)

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^{\alpha}} x^{\alpha-1} e^{-x/\theta}$$

for x ≥ 0. A chi-square distribution with r degrees of freedom is the gamma distribution with $\theta = 2$ and $\alpha = \frac{r}{2}$. The mean of this distribution is r, and its corresponding variance is 2r.

As an example [31], consider a voter preference survey where the null hypothesis is that voters are equally distributed between three candidates. Suppose we randomly sample 150 voters from the population, and 61 voters choose the first candidate, 53 the second, and 36 the third. If we wish to apply a chi-square test to measure how much our empirical data matches our hypothesis, we first compute the chi-square statistic.

$$\chi^2 = \frac{(n_1 - E(n_1))^2}{E(n_1)} + \frac{(n_2 - E(n_1))^2}{E(n_2)} + \frac{(n_3 - E(n_1))^2}{E(n_3)} = \frac{(61-50)^2}{50} + \frac{(53-50)^2}{50} + \frac{(36-50)^2}{50} = 6.52$$

where n_i are the number of voters who selected voter i. With 2 degrees of freedom, the value for $\chi^2_{\alpha=0.05}$ is 5.99. Since the computed value exceeds 5.99, we conclude that at the 0.05 level of significance, there does exist a voter preference for one of the candidates.

A.5 F-tests

The F-distribution is created by two independent chi-square random variables U and V in the following manner:

$$F = \frac{U/r_1}{V/r_2}$$

where r_1 and r_2 are the degrees of freedom of U and V, respectively[28]. This distribution can be used whenever we have two normally distributed sampled populations and randomly (independently)

[28] The symbol F was proposed by George Snedecor as a way of honoring R. A. Fisher.

selected samples from their respective distributions. Often, for convenience, the larger sample variance is placed in the numerator. A discussion of how F-tests are used to test the significance of R^2 values is in AppendixA.6.4.

A.6 Linear Regressions

A linear regression involves obtaining a "best fit" line for a given set of data. The term best fit usually means with respect to a minimum mean-squared error. The order of the regression depends on the number of factors which we think affect the dependent variable. We will begin by introducing a first-order probabilistic model, and then we will proceed to discuss higher-order models.

A.6.1 First-Order Model

A first-order model means that the dependent variable y has only one factor x that it is dependent on as shown in the following equation.

$$y = a_0 + a_1 x + \epsilon$$

where ϵ is a zero-mean random error component. For a given set of data points (x_1, y_1), (x_2, y_2), ..., (x_n, y_n), the goal is to find the values of a_0 and a_1 which minimize the squared error as follows:

$$Error = min_{a_0,a_1} \sum_{i=1}^{n} [y_i - (a_0 + a_1 x_i)]^2$$

The problem can be posed as an overdetermined linear algebra problem (assuming n > 2).

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = B\vec{a}$$

Using the matrix representations, we find that the least squares solution for $\vec{a}$ is

$$\vec{a} = (B^T B)^{-1}(B^T Y)$$

A.6.2 R^2 - The Coefficient of Determination

One way to measure the strength of a factor x in predicting y is to see how much better of a fit using this factor is compared with just using the average value of y. Mathematically, assuming that we have the same n data points as above, and the corresponding values of a_0, a_1, define

$$\bar{y} = \sum_{i=1}^{n} y_i$$

as the mean of observed data points of y and

$$\hat{y}_i = a_0 + a_1 x_i$$

for i=1,2,...,n as the linear, or first-order, estimate of y, for each particular data point. If we define the squared sum of deviations from the mean as

$$SS_y = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

and the mean-squared error of the linear estimate as

$$MSE = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

then the coefficient of determination is defined to be

$$R^2 = 1 - \frac{MSE}{SS_y}$$

which has values between 0 and 1, inclusive. So an R^2 value of 0.7 indicates that the linear estimate has reduced the squared error by 70% over $\hat{y}$ as the predictor. An R^2 of 1 means that the linear estimate is an exact fit of the data, while a value of 0 means that the linear estimate is no better than the average value.

Alternatively, R^2 may also be written as

$$R^2 = \frac{Cov(x,y)^2}{\sigma_x^2 \sigma_y^2}$$

■ A.6.3 Higher-Order Models

Similarly, multi-factor models can be used to represent a dependent variable y. Suppose that we have a p-factor model with independent variables[29] x^1, x^2, ... , x^p, and n sample observations (with n > p). Then, we want to find the values a_i for i = 0, 1, ... , p which minimize

$$Error = min_{a_0,a_1,\ldots,a_p} \sum_{i=1}^{n} [y_i - (a_0 + a_1 x^1 + a_2 x^2 + ... + a_p x^p)]^2$$

with the best-fit equation being

$$y = a_0 + \sum_{j=1}^{p} a_j x^j + \epsilon$$

The solution to the overdetermined linear algebra problem is similar, with the only difference being that

$$B = \begin{bmatrix} 1 & x_1^1 & x_1^2 & \ldots & x_1^p \\ 1 & x_2^1 & x_2^2 & \ldots & x_2^p \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_n^1 & x_n^2 & \ldots & x_n^p \end{bmatrix}$$

In Section 2.2, an example of a multi-factor model will be discussed.

■ A.6.4 F-Test to Determine Significance in Regressions

In a multi-variable regression, we may be interested in whether or not the coefficients to the factors are significant. If we have

$$y = a_0 + \sum_{j=1}^{p} a_j x^j + \epsilon$$

[29]These are p different variables, not p powers of one individual variable. This notation is used to avoid confusion with the notation of x_i above to indicate the i^{th} sample.

then to test the null hypothesis that $a_j = 0 \; \forall$ j, we can apply an F-test, with the F-statistic being

$$F = \frac{\frac{R^2}{p}}{\frac{1-R^2}{n-(p+1)}}$$

where p is the number of factors and n is the number of samples. The degrees of freedom are p and n-(p+1), respectively.

■ B Miscellaneous Details

■ B.1 Hedging Portfolio Forecasts Market Returns

Theorem 3. *The dollar return of the hedging portfolio, S_H, provides the best forecast for the future dollar return of the market.*

Proof. [28] By definition,

$$R^2 = \frac{Cov(S'Q_t, Q_{M(t+1)})^2}{Var(S'Q_t)Var(Q_{M(t+1)})}$$

The numerator is $E(S'Q_tQ'_{t+1}\iota)$, which can be simplified to (1+r)$S'\sigma_{QX}b'\iota$ because $Q_{t+1} = ra + (1+r)bX_t + \tilde{Q}_{t+1}$. Similarly, the only term in the denominator which varies with S is $Var(S'Q_t)$. So, for S to maximize R^2, we need to solve

$$max_S[S'\sigma_{QX}(b'\iota)]$$

subject to the constraint $S'\sigma_{QQ}S = v$, where v is any constant. The constraint ensures that the denominator is constant, while the maximization maximizes the numerator. Up to a scaling constant, the solution is S = $\sigma_{QQ}^{-1}\sigma_{QX}$, which is S_H. ■

■ B.2 Details of Ying's Analysis

Below are some of the details of Ying's [43] work, along with more specifics about his results. Ying defined six different variables

- P(t) ≡ closing price on the t^{th} day
- V(t) ≡ percentage volume of trades on t^{th} day
- ln P(t)
- ln V(t)
- D(ln P(t)) ≡ ln P(t) - ln P(t-1)
- D(ln V(t)) ≡ ln V(t) - ln V(t-1)

and posed the following hypotheses (with results of the statistical tests beneath each hypothesis).

- There is an effect of D(ln P(t)) on D(ln P(t+1))

  Significant at 0.01 level of significance

- There is an effect of D(ln V(t)) on D(ln P(t+1))

  Not significant at the 0.05 level of significance

- There is an effect of ln V(t) on D(ln (P(t+1))

  Significant at the 0.01 level of significance

- There are interaction effects of D(ln P(t)) and D(ln V(t)) on D(ln P(t+1))
- There are interaction effects of D(ln P(t)) and ln V(t) on D(ln P(t+1))
- There are interaction effects of D(ln V(t)) and ln V(t) on D(ln P(t+1))

  Effects of any of the two factors are not significant at the 0.05 level

- There are triple interaction effects of D(ln P(t)), D(ln V(t)), and ln V(t) on D(ln P(t+1))

  The triple interaction effects are significant at the 0.01 level

B.3 Source Code - MixedTraders.m

```
function output = mixedtraders
% Returns expected number of trades versus different probabilities of
% optimists for 100 traders.
%
% WSun 1/1/03

N = 100; output = zeros(93,1);
output(1) = 0.99; % 99/100 chance that there will be exactly 1 trade
for p=0.02:.01:.93
  total = 0;
  q = 1-p;
  for m=1:N
    if (m <= (q*N - 1))
      factor = factorial(round(q*N))/factorial(round(q*N-m+1))* ...
         factorial(N-m+1)/factorial(N);
      firstTerm = factor*(1-(p*N/(N-m+1)))/(N-m+1);
      secondTerm = factor*(p*N/(N-m+1))*(1-(1/(N-m+1)));
    end % if condition
      thirdTerm = 0;
      U = round(max(1,m-1-q*N));
      V = round(min(p*N-1,m-1));
      for r = U:V
       if (m <= (q*N + r + 1))
        product = nchoosek(m-1,r)*nchoosek(N-m+1,round(p*N-r))/ ...
           nchoosek(N,round(p*N))*(p*N-r)/(N-m+1)/(r+1);
        thirdTerm = thirdTerm + product;
       end
      end % for n
      total = total + firstTerm + secondTerm + thirdTerm;
  end % for m
  output(100*p) = total;
end % for p
% Case for p = 1 is 5.1774 from formula for unanimous case
```

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