Joint Image Formation and Anisotropy Characterization in Wide-Angle SAR

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2-D Spotlight-Mode SAR

Objective
- in joint image formation and anisotropy characterization, our goal is to determine \( a_k, \Phi_k \) from the phase-history measurements.

Overcomplete Basis Formulation
- the proposed approach is to expand the scattering function for each scattering center as the sum of an overcomplete set of basis vectors:

\[
\langle f(h) \rangle = \langle f(x, y, h) \rangle \exp \left( -j \frac{2 \pi}{\lambda} \Phi(x, y, h) \right) = \sum_{k} \alpha_k \Phi_k(x, y) \exp \left( -j \frac{2 \pi}{\lambda} \Phi_k(x, y) \right)
\]

Choice of Basis Vectors
- contiguous intervals in aspect angle of non-zero scattering behavior are often observed among scatterers encountered in practice.

Sparsifying Regularization
- the cost function is made differentiable at 0 through the approximation

\[
\left| \frac{\partial J}{\partial \Phi_k(x, y)} \right|^2 
\]

Graph-Structured Interpretation
- the overcomplete basis has an intuitive graph-structured interpretation, given the name basis graph, illustrated below for \( N = 6 \) and rectangular pulse shape, where nodes represent basis vectors and labels to the left indicate anisotropy.

Greedy Graph-Structured Algorithm
- we choose the set of basis vectors such that all widths and shifts in the angular persistence of anisotropy are included:

\[
\text{maximize} \quad \| \alpha_k \|^2 \quad \text{subject to} \quad \| \Phi_k \|^2 \leq \theta \]

Examples
- Quasi-Newton Method — XPatch Data
- we choose the set of basis vectors such that all widths and shifts in the angular persistence of anisotropy are included:

\[
\text{maximize} \quad \| \alpha_k \|^2 \quad \text{subject to} \quad \| \Phi_k \|^2 \leq \theta \]

Graph-Structured Algorithm — Synthetic Data
- there are \( P = 7 \) spatial locations and \( N = 1541 \) angles over a 110º aperture

Graph-Structured Algorithm — Backhoe Data
- there are \( P = 7 \) spatial locations and \( N = 1541 \) angles over a 110º aperture

Heuristics and Stopping Criteria
- true anisotropy finest than current guiding graph:
  - bottom row coefficients non-zero
  - slide guiding graph down
  - guide left or right weighted average of bottom row coefficients
  - true anisotropy coarse than current guiding graph:
    - top row coefficient non-zero
    - slide guiding graph up
  - true anisotropy inside current guiding graph:
    - true coefficients non-zero
    - do not move guiding graph — stop

Strategy: Guided Depth-First Search
- follow one path down from the root node with the path based on a heuristic.

The above diagram illustrates the overcomplete basis for \( N = 8 \) — dots indicate nonzero entries and spaces represent zero-valued entries; any pulse shape may be used for the basis vectors, e.g. rectangular; Hamming window, triangular, raised triangle, windowed Gaussian.

The diagram below illustrates the iterations of a search where true anisotropy is represented by the node with the X

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