Walk-Sums and Gaussian BP (#210)

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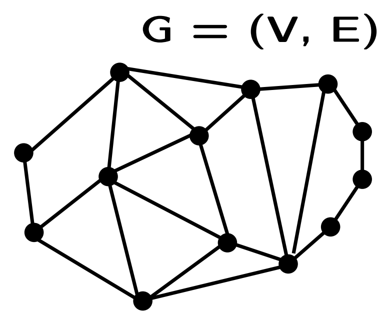
Walk-sum framework for Gaussian inference, new sufficient conditions for BP convergence.

Information form of the Gaussian density:

$$p(x_1, ..., x_n) \propto \exp\{-\frac{1}{2}x^T J x + h^T x\}$$

J is sparse:



Define edge weights $\rho_{ij} = -J_{ij}/\sqrt{J_{ii}J_{jj}}$.
Given a walk w in G, let $\rho(w) = \prod_{(i,j)\in w} \rho_{ij}$.

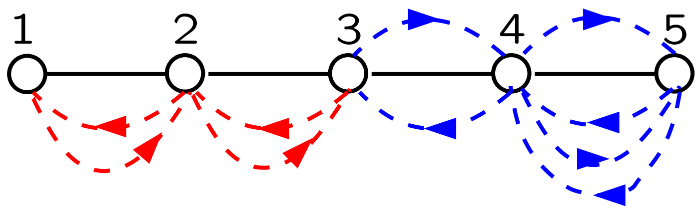
Walk-summable if spectral radius $\varrho(|R|) < 1$, R is matrix of edge weights.*

$$\text{cov}(x_i, x_j) = \sum_{w:i\to j} \rho(w), \;\; \text{mean}(x_i) = \sum_{w:*\to i} h_* \rho(w)$$

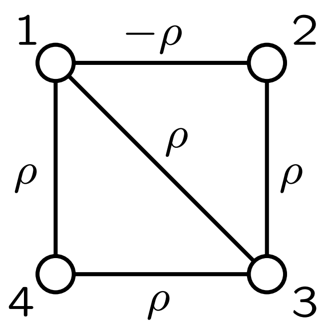
*Includes trees, attractive models and diagonally dominant models.

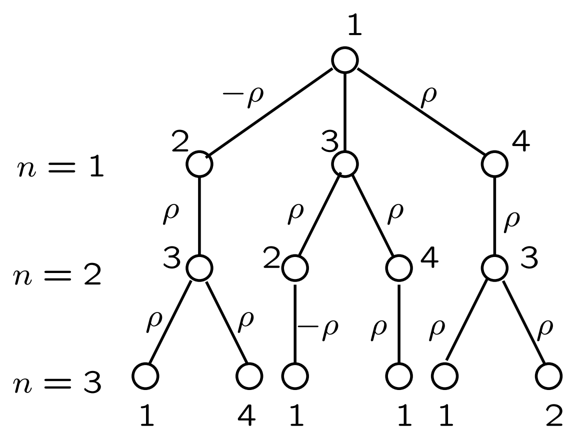
Walk-Sum View of Belief Propagation

BP in trees $\equiv$ recursive walk-sum calculation:



Loopy BP on G is equivalent to computing exact walk-sums in the computation tree:





WS on G $\Rightarrow$ BP converges: means correct, variances $\rightarrow$ sums over *backtracking* walks.

A tighter condition is WS on the comp. tree:
(i) $\varrho_\infty = \lim_{n\to\infty} \varrho(|R_n|) \le \varrho(|R|)$.
(ii) $\varrho_\infty < 1 \Rightarrow$ BP variances converge.
(iii) $\varrho_\infty > 1 \Rightarrow$ invalid computation tree.