Recursive Composition Inference for Force Aggregation

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Abstract - *This paper describes recursive composition inference techniques (identified with inference on a Bayesian network) to efficiently and optimally reason about military units from partial observations of constituent vehicles. This model-based approach requires statistical models of the composition of military units as well as of the processes of vehicle detection and observation. Given these statistical models the Bayesian network both infers unit-type and refines vehicle-type conditioned upon partial observations of the detected vehicles. Monte Carlo experiments are performed to provide performance estimates under variable conditions. The main innovation of this approach is the recursive decomposition of composition hypotheses to efficiently infer between vehicle-level hypotheses and unit-level hypotheses. The primary benefit of this work is that it provides a means for reasoning about aggregates of objects which is simple, robust and rigorous.*

Key Words: force aggregation, situation refinement, Bayesian inference, Bayesian networks.

1. Introduction

Military units organize themselves in hierarchical structures to enable efficient deployment, operation, and command and control. Determining the hierarchical structure of an opposing force is critical to determining its capability, threat, and ultimately its intent. Sensor systems can detect and take measurements on individual entities, such as vehicles and installations. These measurements can be used to infer the class of individual entities. However, few collection assets provide direct measurements on the hierarchical force structure of units that the entities comprise. Consequently, it is desirable to develop the capability to draw inferences on the hierarchical structure of military units based on inferences and measurements of individual entities and sub units.

This paper describes a technique for assessing the relative merit of force aggregate hypotheses from partial observations of a set of entities. That is, given partial observations of entities that comprise military units, the technique draws inferences about the type of military unit that is present. Furthermore, drawing inference about the type of military unit provides contextual information that enables improved inference about the type of individual vehicles.

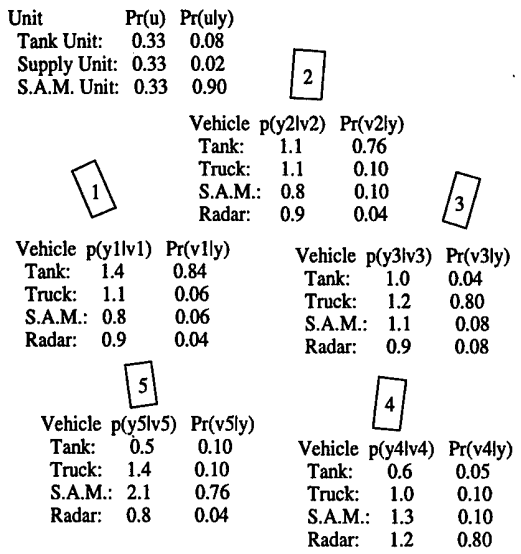

Figure 1. Illustration of a notional estimation scenario.

2. Model Specification

The force aggregation problem solved in this paper may be stated as follows. The available data are partial observations of a military unit in the form of target reports for detected targets. Each target report provides partial information as to the classification of the detected target. The objective is then twofold. First, fuse these target reports to infer unit-type. Second, exploit context to refine vehicle-type. Figure 1 provides a notional scenario to illustrate this problem. The solution described in this paper operates strictly by reasoning about the composition of military units.

In order to perform probabilistic inference we must formulate a probabilistic model for the underlying process being observed. The model we will employ in this paper is developed below. This model consists of three tiers: the composition model of the military units; the detection model of the vehicle detector; and the measurement model of the process by which the type of each detected vehicle is partially observed. A diagram of the overall structure of the model is provided in Figure 2.

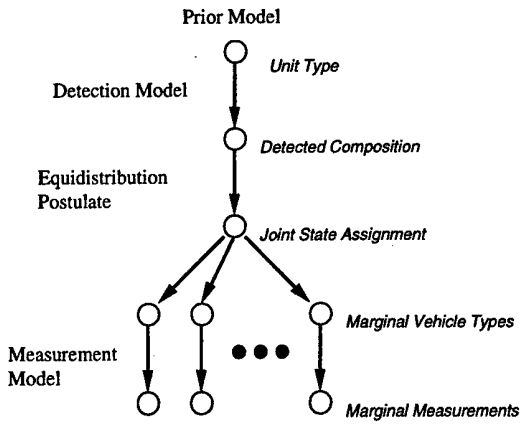

Figure 2. Diagram of Bayes net used to model fusion problem. Note, the joint-state assignment node is replaced by a recursive binary tree model in section 3.2 (Figure 5).

2.1 The Composition Model

The composition model specifies what unit-types may occur, the composition of these units and the prior probability of the occurrence of each unit-type. Unit-type is modeled by the random variable U assuming the values $u=1,\ldots,q$ with prior probability $\Pr(U=u)$. Vehicle-type is modeled by the random variable V assuming the values $v=1,\ldots,r$. The composition of each unit-type is then specified as the number of instances of each vehicle-type present within the unit. The number of vehicles of type v present within units of type u is denoted $n(v;u)$ for $v=1,\ldots,r$ and $u=1,\ldots,q$. Hence, specification of the composition model consists of an r x q matrix of non-negative integers (vehicle counts) and a q-length vector of prior probabilities (which sum to unity). Note, this composition model could easily be extended to also model uncertainty in the composition of a given unit-type. However this extra layer of uncertainty is omitted in this paper.

2.2 The Detection Model

The detection model is a statistical model of the process of detecting military vehicles in the signal-level data generated by some sensor. There are three sources of uncertainty that tend to obscure the identity of the military unit: undetected vehicles, extraneous clutter vehicles and false alarms. The detection model is included so as to provide robustness against these anticipated ATR operating conditions. The occurrence of these failures of detection are statistically modeled by the Bernoulli probability of detection as a function of vehicle-type $P_D(v)$ the Poisson clutter rate also as a function of vehicle type $\lambda_C(v)$ and the Poisson false-alarm rate λ_{FA}. A diagram illustrating the cumulative effect of these three detection processes is shown in Figure 3. The "null hypothesis" $v=0$ is introduced to denote false alarms.

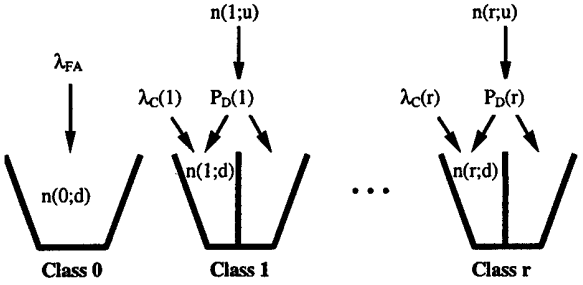

Figure 3. Diagram of the detection model.

As indicated in Figure 2, the role of the detection model is to specify the transition probabilities from the unit-type (of known composition) to the detected composition (the composition of the set of detected vehicles). However, these transition probabilities depend upon the number of detected vehicles n (implicitly part of our observation of the military unit). Once the number of detections is received the transition probabilities may then be computed as shown below. First, due to the independence of the various detection processes this transition probability from unit composition u to detected composition d is separable by vehicle-type (the number of objects accumulated in each of the left-hand bin partitions of Figure 3 are conditionally independent given the unit-type).

$$(1) \qquad \Pr(d \mid u) = \Pr(n(0;d)) \prod_{v=1}^{r} \Pr(n(v;d) \mid n(v;u))$$

Note, the notation $n(v;d)$ indicates the number of detected vehicles of class v. The probability of the number of detected vehicles of each type v is then computed as

$$(2) \quad \Pr(n(v;d) \mid n(v;u)) = \sum_{k=0}^{n(v;u)} b(k; n(v;u), P_D(v)) \times p(n(v;d) - k; \lambda_C(v))$$

where $b(k;n,p)$ are the binomial probabilities

$$(3) \qquad b(k;n,p) = \begin{cases} \binom{n}{k} p^k (1-p)^{k-n}, & k \in [0,n] \\ 0, & k \notin [0,n] \end{cases}$$

and $p(k;\lambda)$ are the Poisson probabilities

$$(4) \qquad p(k;\lambda) = \begin{cases} \lambda^k e^{-\lambda}/k!, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

The number of false alarms generated by the detector is also modeled by (4). These transition probabilities will be used in Section 3 to infer between unit-type and detected composition.

2.3 The Measurement Model

The measurement model is a statistical model of the process of observing a detected vehicle so as to produce a measurement of that vehicle. Each vehicle measurement y is modeled as being a random function of the observed vehicle's type v. This random function is statistically characterized by a measurement probability density function $p(y|v)$ for each of the possible vehicle types $v=1,\ldots,r$. Also, if false-alarms are modeled then a measurement model must be provided for those as well.

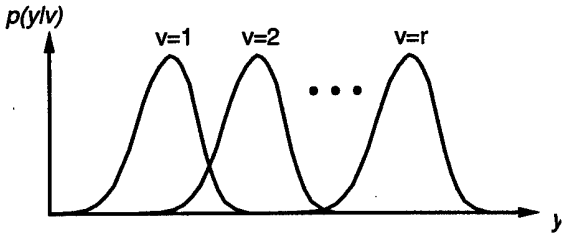

Figure 4. Plot of a notional measurement model.

3. Composition Inference Algorithms

The statistical models specified above provide the basis for implementing rigorous probabilistic inference algorithms for the optimal estimation of both unit-type and vehicle-type from partial observations of the detected vehicles. The data available to the inference algorithms are the number of detected vehicles n as well as a measurement of each of these vehicles y_k for $k=1,\ldots,n$. For the remainder of this paper y will denote the set of all such observations. Given these data the objective of the inference algorithm is then twofold. First, calculate the unit-type probabilities conditioned upon all observations $\Pr(U=u \mid y)$ for $u=1,\ldots,q$. Second, calculate the vehicle-type probabilities conditioned upon all observations $\Pr(V_k=v \mid y)$ for $k=1,\ldots,n$ and $v=0,1,\ldots,r$. These conditional probabilities then provide the basis for rendering optimal marginal estimates of the unit-type of the observed unit and of the vehicle-type of the detected vehicles. These inference calculations are outlined below.

To infer unit-type from vehicle observations, first the measurement data are pre-conditioned under the measurement model. This involves computation of the measurement likelihood conditioned upon vehicle-type $p(y_k \mid V_k=v)$ for each vehicle $k=1,\ldots,n$ as a function of vehicle-type $v=0,1,\ldots,r$. At this point the measurements themselves may be discarded as these likelihoods are sufficient statistics of those measurements for the purposes of inferring both unit-type and vehicle-type.

Next, the composition of detected vehicles is inferred by either of the methods discussed below in sections 3.1 and 3.2. This involves calculation of the measurement likelihood $p(y|d)$ as a function of detected composition d. The fundamental postulate underlying either method is the Equidistribution Postulate stated below.

Equidistribution Postulate: *Conditioned upon the composition of some set of vehicles, all joint-state assignments of vehicle-type to those vehicles consistent with the composition constraint are equally probable.*

To state this mathematically the set of all joint-state assignments (of vehicle classes to vehicles) consistent with a specified detected composition hypothesis d is denoted as $\Omega(d)$. Hence (according to the Equidistribution Postulate) the conditional probability of an assignment a conditioned upon a composition d is given by

$$(5) \qquad \Pr(a \mid d) = \begin{cases} 1/|\Omega(d)|, & a \in \Omega(d) \\ 0, & a \notin \Omega(d) \end{cases}$$

where $|\Omega(d)|$ is the degeneracy of the composition (the number of assignments of vehicle types to vehicles consistent with the composition) given by the multinomial coefficients.

$$(6) \qquad |\Omega(d)| = \binom{n(d)}{n(1;d),\ldots,n(r;d)}$$

Once the measurement likelihood as a function of the detected composition has been inferred, the measurement likelihood conditioned upon unit-type may then be inferred employing the transition probabilities from unit composition to detected composition as computed under the detection model (1-4).

$$(7) \qquad p(y \mid u) = \sum_{d \in D(n,r+1)} p(y \mid d) \Pr(d \mid u)$$

This sum is taken over the set $D(n,r+1)$ of all possible distributions of the n detected vehicles into the $r+1$ vehicle-classes (included the false-alarm class). Finally, the unit-type probabilities are computed according to Bayes rule from the measurement likelihoods $p(y \mid u)$ and the prior model $\Pr(u)$.

$$\Pr(u \mid y) = \frac{p(y \mid u)\Pr(u)}{p(y)} \tag{8}$$

The denominator $p(y)$ is the likelihood of all measurements which simply normalizes the relative probabilities computed by the numerator.

$$p(y) = \sum_{u=1}^{q} p(y \mid u)\Pr(u) \tag{9}$$

This is the basic structure of the unit-type inference algorithm (except for the composition inference step which is discussed in sections 3.1 and 3.2).

To refine vehicle-type the detected composition (implicitly conditioned upon the number of detected vehicles) is inferred from unit type using the prior model and the detection model.

$$\Pr(d) = \sum_{u=1}^{q} \Pr(d \mid u)\Pr(u) \tag{10}$$

Then vehicle-type probabilities $\Pr(V_k=v \mid y)$ for each vehicle $k=1,\ldots,n$ as a function of vehicle-type $v=0,\ldots,r$ are inferred from the detected composition probabilities $\Pr(d)$ and the Equidistribution Postulate. This calculation is deferred to section 3.2.

3.1 Brute Force Composition Inference

Before developing the recursive composition inference techniques in the next subsection a simpler brute force approach is considered. This calculation operates by performing inference with respect to the set of all possible joint-states of the detected vehicles. Due to the independence of the vehicle measurements, the measurement likelihood conditioned upon the joint state assignment is simply the product of the marginal measurement likelihoods conditioned upon the respective marginal vehicle types.

$$p(y \mid a) = \prod_{k=1}^{n} p(y_k \mid v_k) \tag{11}$$

The likelihood of all measurements conditioned upon the detected composition may then be computed from the assignment likelihoods (11) and the transition probabilities (5).

$$\begin{aligned} p(y \mid d) &= \sum_{a} p(y \mid a)\Pr(a \mid d) \\ &= \frac{1}{|\Omega(d)|} \sum_{a \in \Omega(d)} p(y \mid a) \end{aligned} \tag{12}$$

This likelihood computation is performed for every possible distribution d of the n detected vehicles into the $r+1$ vehicle-classes. Once this likelihood function has been computed the unit-type may then be inferred as described previously (7-10).

Under this approach the likelihood calculation (11) must be computed for each of the $(r+1)^n$ possible joint-state assignments of the detected vehicles such that the complexity of these calculations is $O(n(r+1)^n)$. This prohibitive computational complexity is the motivation for the recursive composition inference techniques developed in the next section.

3.2 Recursive Composition Inference

In this section we develop recursive composition inference techniques which offset the computation burden of the simplistic approach described above. This technique avoids considering the set of $(r+1)^n$ joint-states of the detected vehicles by recursively partitioning this set of vehicles into half-sets and inferring between the composition of the halves and the composition of the whole. Figure 5 illustrates the Bayesian structure of this decomposition. Note that Figure 5 is an alternate expansion of the middle three tiers of the Bayes net shown in Figure 2 (replacing the "brute force" inference technique depicted there).

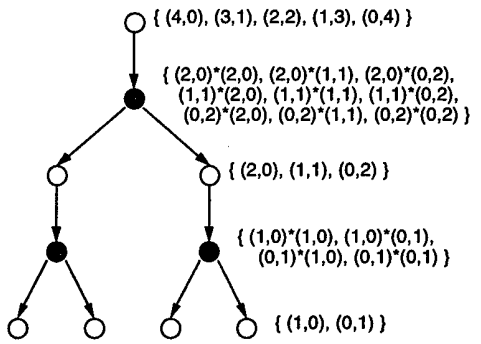

Figure 5. Bayes net to recursively infer composition. For this example there are four vehicles which submit to two classifications.

There are two types of nodes depicted in Figure 5. The state-space of a white node corresponds to the set

of all possible compositions of the vehicles beneath that node (the leaf nodes). For example, the states of a leaf node correspond directly to the vehicle-type of the associated vehicle, also the states at the root node correspond to the composition of all detected vehicles. In order to statistically relate the composition of a set of vehicles to the composition of its two subsets the grey nodes are introduced such that the state of the subtrees are conditionally independent given the state of the grey node. This is achieved simply by choosing the states of the grey node to be the joint-state of its two white-node children. Hence the states of a grey node essentially represent hypotheses as to how the composition of the whole is distributed between the halves. The composition of the two halves may then be considered independently.

The transition probabilities from a white-node state d_T (specifying the total composition of the n_T vehicles contained within that subtree) to a grey-node state (d_L, d_R) (specifying the respective composition of the n_L and n_R vehicles in the left and right subtrees) are determined by the Equidistribution Postulate.

$$\Pr(d_L, d_R \mid d_T) = \begin{cases} \dfrac{|\Omega(d_L)| \times |\Omega(d_R)|}{|\Omega(d_T)|}, & d_L + d_R = d_T \\ 0, & d_L + d_R \neq d_T \end{cases} \tag{13}$$

The requirement that $d_L+d_R=d_T$ indicates that the sub-compositions must be consistent with the total composition which is to say that $n(v;d_L)+n(v;d_R)=n(v;d_T)$ for $v=0,\ldots,q$. The transition probabilities from grey nodes to white nodes are trivial since the state of the grey node is just the joint-state of its two white-node children.

Below the standard Bayesian inference algorithms (see [2]) are adapted to take advantage of the sparsity of the transition probabilities described above (for the sake of efficient computation) as well as to exploit the special structure of (13) (to simplify the prediction operations).

Recursive Composition Inference

The objective of recursive composition inference is to propagate the information contained in the measurements at the leaves of the tree upwards fusing this information at each node s to yield statistics concerning the set of all measurement on that subtree y_s. Rather then propagating the traditional likelihood function $p(y_s|d_s)$ we instead propagate the likelihood function weighted by the degeneracy of the composition $|\Omega(d_s)|$ which is denoted by $\lambda_s(d_s)$.

$$\lambda_s(d_s) = p(y_s \mid d_s) \times |\Omega(d_s)| \tag{14}$$

The advantage of propagating these functions is that the degeneracy weights cancel with the transition probabilities (13) such that prediction and merging of these messages from the two subtrees simplifies to the formula below.

$$\lambda_T(d_T) = \sum_{\substack{d_L, d_R \\ d_L + d_R = d_T}} \lambda_L(d_L) \times \lambda_R(d_R) \tag{15}$$

Here the sum is taken over all possible compositions of each of the two subtrees (d_L and d_R respectively) under the constraint that their composition sum to the composition d_T. An algorithm for accomplishing this calculation is outlined below.

```
Inference Code
Inputs: λ_L, λ_R
Outputs: λ_T
Initialize λ_T
for d_T ∈ D(n_T, r+1)
  λ_T(d_T) = 0
end
Accumulate λ_T
for d_L ∈ D(n_L, r+1)
  for d_R ∈ D(n_R, r+1)
    let d_T = d_L + d_R
    λ_T(d_T) = λ_T(d_T) + λ_L(d_L) × λ_R(d_R)
  end
end
```

This code fragment is recursively applied to the Bayes net starting at the bottom of the tree and propagating statistics up the tree until the root node is reached. This recursion is initialized at the leaves according to the measurement model (note, there is no degeneracy at the leaf nodes). At the root node of the tree the likelihood function $p(y|d)$ is recovered from $\lambda(d)$ by dividing by the degeneracy function (6) in accordance with (14).

Once these measurement likelihoods are computed the unit-type may then be inferred by formulas (7-9). These probabilities may then be used to render an optimal marginal estimate of unit-type.

Recursive Composition Refinement

The objective of recursive composition refinement is to propagate prior information down the tree as well as to redistribute the information passed up from each subtree to the other subtree so as to compute statistics

at each node s conditioned upon all measurements not on that subtree $\bar{y}_s = y \setminus y_s$. Rather than propagating the traditional probability mass function $\Pr(d_s \mid \bar{y}_s)$ we instead propagate a function $\pi_s(d_s)$ proportional to the probability mass function weighted by the inverse of the degeneracy $|\Omega(d_s)|$.

$$(16) \qquad \pi_s(d_s) \propto \frac{\Pr(d_s \mid \bar{y}_s)}{|\Omega(d_s)|}$$

Again the degeneracy factor is included so as to cancel with the factors of the transition probabilities (13). The prediction of this function from parent node T to child nodes L and R then simplifies to the formula below.

$$(17) \qquad \begin{aligned} \pi_L(d_L) &= \sum_{d_R \in D(n_R, r+1)} \lambda_R(d_R) \times \pi_T(d_L + d_R) \\ \pi_R(d_R) &= \sum_{d_L \in D(n_L, r+1)} \lambda_L(d_L) \times \pi_T(d_L + d_R) \end{aligned}$$

An algorithm for accomplishing these calculations is outlined below. Note, this algorithm requires that the composition inference has already been executed such that the statistics $\lambda_s(d_s)$ are available for each node s.

```
Refinement Code
Inputs: π_T, λ_L, λ_R
Outputs: π_L, π_R
Initialize π_L
for d_L ∈ D(n_L, r+1)
  π_L(d_L) = 0
end
Initialize π_R
for d_R ∈ D(n_R, r+1)
  π_R(d_R) = 0
end
Accumulate π_L and π_R
for d_L ∈ D(n_L, r+1)
  for d_R ∈ D(n_R, r+1)
    let d_T = d_L + d_R
    π_L(d_L) = π_L(d_L) + λ_R(d_R) × π_T(d_T)
    π_R(d_R) = π_R(d_R) + λ_L(d_L) × π_T(d_T)
  end
end
```

By applying this code fragment to the Bayes net in a recursive down-sweep manner (start at the root node, compute statistics of children and recurse on subtrees) the statistics $\pi_s(d_s)$ are computed at every node s. This recursion is initialized at the root node according to (16) from the prior composition statistics (10) (conditioned upon the number of detections) and the degeneracy function (6). The final probability computation conditioned upon all observations as a function of the state of the node $\Pr(d_s|y)$ is computed by merging these statistics $\pi_s(d_s)$ with those computed during the composition inference algorithm $\lambda_s(d_s)$.

$$(18) \qquad \Pr(d_s \mid y) = \frac{\lambda_s(d_s) \times \pi_s(d_s)}{p(y_s \mid \bar{y}_s)}$$

The likelihood ratio shown in the denominator is simply the norm of the relative likelihoods computed by the numerator.

$$(19) \qquad p(y_s \mid \bar{y}_s) = \sum_d \lambda_s(d) \times \pi_s(d)$$

Formulas (18) and (19) are used to compute the refined vehicle-type probabilities which may then by used to render optimal marginal estimates of vehicle-type.

While the complexity of these recursive calculations is much improved over the brute-force approach, the complexity of the recursive calculations nevertheless grows rapidly with n ($O(n^2)$ for r=2). For this reason we briefly comment on the possibility of employing hypothesis pruning to limit the computational complexity of the recursive approach to $O(n)$ (at the expense of performing sub-optimal inference). This is accomplished by implementing a pruning operation in the composition inference upsweep such that only those N compositions maximizing the function $\lambda_s(d)/|\Omega(d)|$ (a lower-bound approximation of $p(y|d)$) are retained.

4. Monte Carlo Performance Estimation

In this section Monte Carlo simulation techniques are used to characterize the performance of the inference techniques developed above under various circumstances. In this section we characterize the difficulty of the problem by the parameters listed below. Unless otherwise stated, these parameters are chosen as follows.

n =3	number of vehicles per unit
q =2	number of unit-types
r =4	number of vehicle types
P_D =0.9	probability of detection
λ_C =0.25	vehicle clutter rate
λ_{FA} =1	detector's false alarm rate
J =4	divergence between vehicle-types

For each Monte Carlo trial we randomly select the q unit composition models by sampling the vehicle-type of each of the unit's n vehicles uniformly from the r vehicle-types. The true unit-type is then selected from among these q unit-types with uniform probability. The detection model is then applied to the composition of the true unit-type to simulate the number of detected vehicles of each target class (including the false-alarm class). Finally, the measurement likelihood function is simulated for each detected vehicle conditioned upon the vehicle-type as described below.

The divergence J is an information-theoretic measure (see [3]) of how easily two hypotheses 0 and 1 may be discriminated on average given a measurement y.

$$(20) \qquad J(0,1) = \int_y (p(y\,|\,1) - p(y\,|\,0)) \log \frac{p(y\,|\,1)}{p(y\,|\,0)}\, dy$$

For Gaussian measurement models having a common covariance P and separation between the means Δ the divergence is $J=\Delta^T P^{-1}\Delta$. In this case, the distribution of the log-likelihood-ratio $L(1|0)=\log(p(y|1)/p(y|0))$ is normally distributed having variance $J^{1/2}$ and mean values $-J/2$ and $+J/2$ under the respective hypotheses 0 and 1. Hence, the likelihood function may be simulated (up to an undetermined scale factor) under hypothesis 1 as ($\lambda(0)=e^L$, $\lambda(1)=1$) for $L\sim N(-J/2, J^{1/2})$. We take the liberty of generalizing this model to the N-ary hypothesis case by independently sampling L for each confusor class (all vehicle-types except the true vehicle-type). The goal here is to simply characterize the uncertainty of the measurement process by the single parameter J in order to facilitate generic performance estimation.

Given the simulated target-reports we may then apply the inference techniques of section 3 to infer the hidden unit-type and the hidden vehicle-type of detected vehicles. These estimates are then compared to the true values and estimates of the probability of correct-classification are accumulated for both unit-level and vehicle-level estimates. Unless otherwise stated, each of the performance plots provided below are based upon 1000 independent iterations of the above simulation for each data-point.

First the performance of unit-type estimation is examined by plotting the probability of correct-classification as a function of the divergence between measurement models (Figure 6).

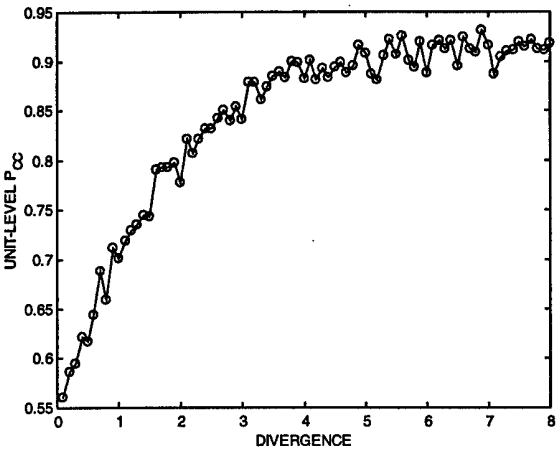

Figure 6. Plot of unit-type P_{CC} vs. J.

Likewise, the performance of vehicle-type estimation is measured by the vehicle-level probability of correct-classification plotted as a function of the divergence between measurement models (Figure 7). Here we plot the estimation performance for both the unrefined estimates (each vehicle's type is estimated so as to maximize the likelihood of just that vehicles measurement) and the refined estimates (conditioned upon all measurements).

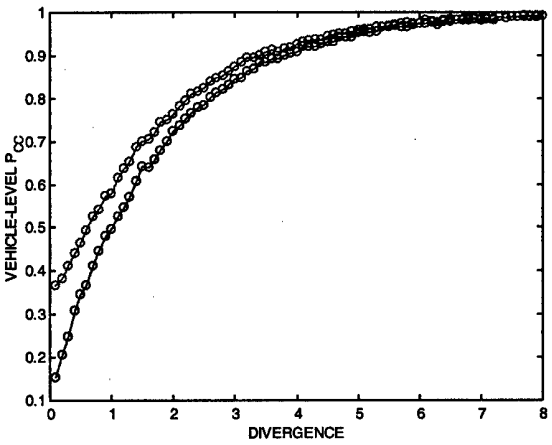

Figure 7. Plot of vehicle-type P_{CC} for refined (top) and unrefined estimates (bottom) vs. J.

It is also interesting to examine the unit-level classification performance as a function of the uncertainty of the detection process. Hence the unit-level P_{CC} is plotted as a function of P_D (Figure 8) and the clutter rate λ_C (Figure 9)

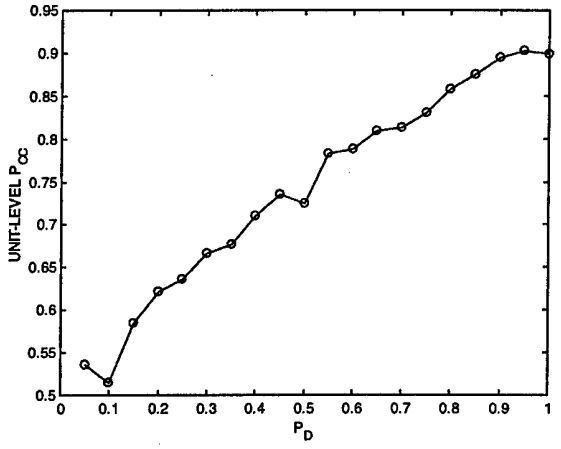

Figure 8. Plot of unit-type P_{CC} vs. P_D of detector.

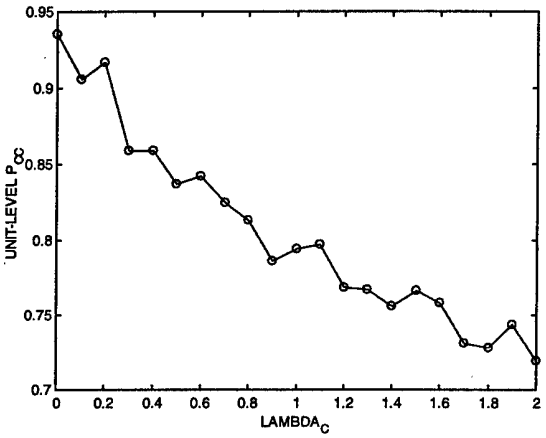

Figure 9. Plot of unit-type P_{CC} vs. clutter-rate λ_C.

Finally, the trade-off between classification performance and run-time introduced by hypothesis pruning is examined in Figure 10. This is a plot of P_{CC} vs. average run-time traced out by varying the maximum number of hypotheses control parameter N from 1 to 19 (both performance and run-time level off for larger values of N). For this simulation we chose the parameters q=2, n=3, r=6, P_D =1, λ_C=0, λ_{FA}=0, and J=1. Each of these data points represent 10000 Monte Carlo trials.

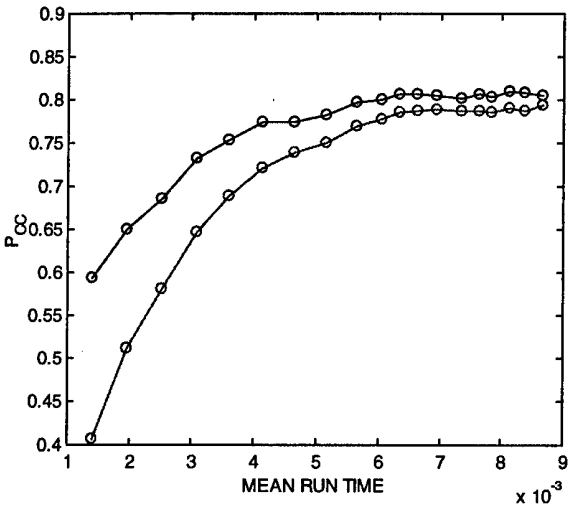

Figure 10. Plot of P_{CC} (upper curve is unit-type, lower-curve is vehicle-type) vs. average run-time (ms) traced out by varying the pruning control parameter N from 1 to 19.

5. Example Application

This section briefly introduces the application of the inference techniques developed in this paper to hierarchical force structures. The key idea here is that the same inference techniques used to infer unit-type from observations of the unit's constituent vehicles may also be employed to infer the type of a complex unit (consisting of multiple subunits) from knowledge of its subunits. This concept may be applied recursively to analyze an arbitrarily complex hierarchical force structure. This concept is illustrated below (Figure 11).

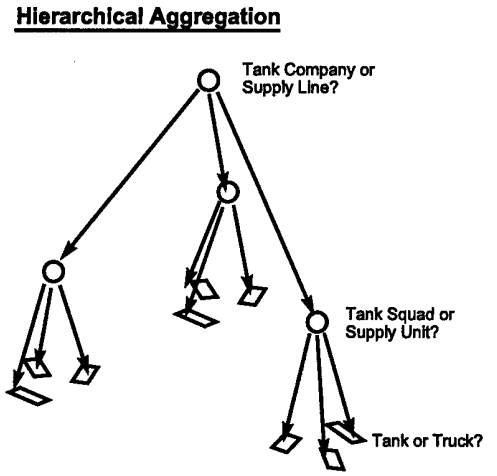

Figure 11. Illustration of hierarchical force inference.

An example of this technique is shown if Figures 12 and 13. Figure 12 depicts a simulated hierarchical force deployment. Partial observations of the vehicles within this hierarchy were simulated (J=1, P_D=1, λ=0) and provided to the hierarchical estimator. The resulting estimates are shown in Figure 13. All entities were classified correctly except that the *TelSquadConvoy* was classified as *TelSquadDefend.* This is because the composition-based inference cannot distinguish between the different configurations of a unit.

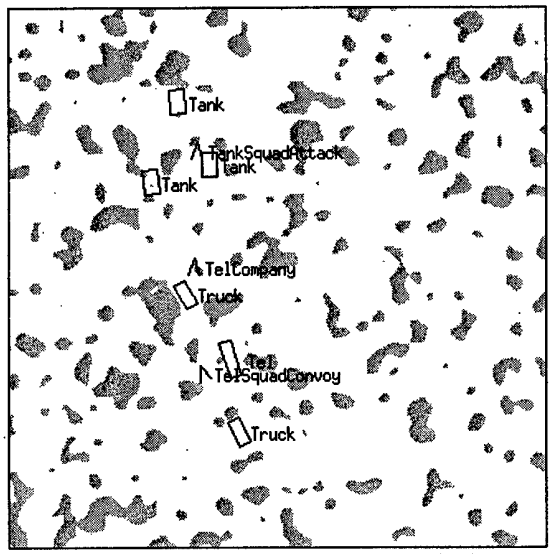

Figure 12. A simulated hierarchical force deployment.

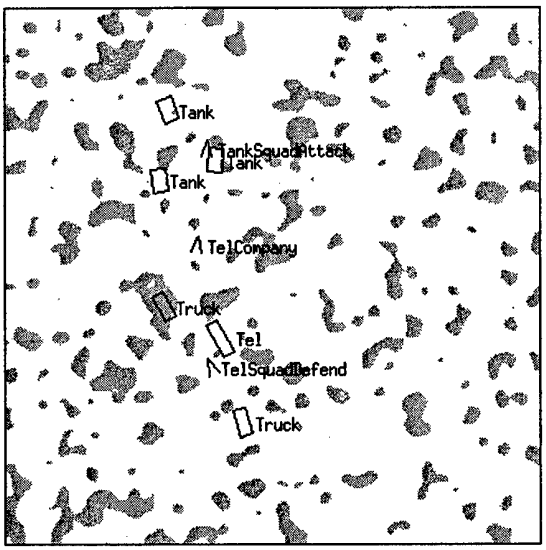

Figure 13. Estimates resulting from inference within this force structure inference.

6. Conclusions

In this paper we have described a simple, robust and rigorous technique for inferring the unit-type of a partially observed military unit as well as refining the vehicle-types of the detected vehicles. The simplicity of the model insures its generality, robustness and ease of model identification. Monte Carlo simulations demonstrate the utility and robustness of these techniques. In future research we plan to extend these techniques to model the spatial deployment of military units, to utilize terrain information, and to provide for automatic clustering of detected vehicles by searching for the clustering which maximizes the likelihood of the observations.

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