Strategic Supply Function Competition with Private Information

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Abstract

A Bayesian supply function equilibrium is characterized in a market where firms have private information about their uncertain costs. It is found that with supply function competition, and in contrast to Bayesian Cournot competition, competitiveness is affected by the parameters of the information structure: supply functions are steeper with more noise in the private signals or more correlation among the costs parameters. In fact, for large values of noise or correlation supply functions are downward sloping, margins are larger than the Cournot ones, and as we approach the common value case they tend to the collusive level. Furthermore, competition in supply functions aggregates the dispersed information of firms (the equilibrium is privately revealing) while Cournot competition does not. The implication is that with the former the only source of deadweight loss is market power while with the latter we have to add private information. As the market grows large the equilibrium becomes competitive and we obtain an approximation to how many competitors are needed to have a certain degree of competitiveness.

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1. Introduction

Competition in supply functions has been used to model several markets, in particular the spot market for electricity but also management consulting or airline pricing reservation systems. The models considered typically do not allow for private information.[1] Private information on costs is a relevant situation in many instances where it is not realistic to assume that there is common knowledge on costs. Instead each firm has an estimate of its own costs and uses it, together with whatever public information is available, to make inferences about the costs of rivals. In this paper we study supply function competition when firms have private information about costs and compare it with Cournot competition, a leading modeling contender. Our aim is to explore the impact of private information on price-cost margins, competitiveness, and welfare.

Competition in supply schedules has been studied in the absence of uncertainty by Grossman (1981) and Hart (1985) showing a great multiplicity of equilibria. A similar result is obtained by Wilson (1979) in a share auction model. Back and Zender (1993) and Kremer and Nyborg (2004) obtain related results for Treasury auctions. Some of the equilibria can be very collusive.[2] Klemperer and Meyer (1989) show how adding uncertainty in the supply function model can reduce the range of equilibria and even pin down a unique equilibrium provided the uncertainty has unbounded support.[3] In this case the supply function equilibrium always lies between the Cournot and competitive (Bertrand) outcomes. Kyle (1989) introduces private information into a double auction for a risky asset of unknown liquidation value and derives a unique symmetric linear Bayesian equilibrium in demand schedules when traders have constant absolute risk aversion, there is noise trading, and uncertainty is normally distributed.

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1 Exceptions are the empirical papers of Hortaçsu and Puller (2006) and Kühn and Machado (2004) in electricity.

2 Back and Zender (2001) and LiCalzi and Pavan (2005) show how the auction can be designed to limit those collusive equilibria.

3 In a linear-quadratic model this is a linear equilibrium.

The modeling strategy in this paper is to consider linear-quadratic payoffs coupled with an affine information structure, which admits common or private values, that yields a unique symmetric linear Bayesian supply function equilibrium (LBSFE). We do not need to introduce noise in the system. The characterization of a linear equilibrium with supply function competition when there is market power and private information needs some careful analysis in order to model the capacity of a firm to influence the market price at the same time that the firm learns from the price. Kyle (1989) pioneered this type of analysis in a financial market context introducing noise trading in order to prevent the market from collapsing.

It is found that there is a unique LBSFE except in the pure common value case. This equilibrium is privately revealing. That is, the private information of a firm and the price provide a sufficient statistic of the joint information in the market. This means in particular that the incentives to acquire information are preserved despite the fact that the price aggregates information. We do not examine possible nonlinear equilibria. Linear equilibria are tractable, in particular in the presence of private information, and have desirable properties like simplicity.

In the linear equilibrium supply functions are upward sloping provided that the informative role of price does not overwhelm its traditional capacity as index of scarcity. This happens when costs shocks are not very correlated and information precision not too low. In this case an increase in the correlation of cost parameters or in the noise in private signals makes supply functions steeper. Firms are more cautious when they see a price raise since it may mean that costs are high. The market looks less competitive in those circumstances as reflected in increased price-cost margins. Ignoring private cost information with supply function competition may therefore overestimate the slope of supply. This is not the case with Cournot competition, where competitiveness and the margin are not affected by the information parameters. When the information role of the price dominates its index of scarcity capacity supply functions slope downwards and margins are larger than the Cournot ones. This is in contrast of the results in Kyle (1989), and also in Wang and Zender (2002), where *demand* schedules always slope downwards.

The result implies, in particular, that –in contrast with Klemperer and Meyer (1989)– margins with supply function competition can be higher than the Cournot level. More surprisingly perhaps, as we approach the common value case margins tend to the collusive level. This happens at the unique linear equilibrium only for informational reasons and not because of the existence of a vast multiplicity of equilibria. Relaxation of competition due to adverse selection in a common value environment is also obtained in Biais et al. (2000).

A welfare optimal allocation can be implemented by a price-taking Bayesian supply function equilibrium. This is so since at a LBSFE there is a deadweight loss only because of market power since the equilibrium is privately revealing. Typically the deadweight loss increases as we approach the common value case as long as signals are noisy. The welfare evaluation of the LBSFE is in marked contrast with the Cournot equilibrium in the presence of private information. The reason is that the LBSFE aggregates information and therefore, as stated, there is only a deadweight loss due to market power but not due to private information. The result is that in a large market with supply function competition there is no efficiency loss (in the limit) and the order of magnitude of the deadweight loss is $1/n^2$ where n is the number of firms (and the size of the market as well). This is also the rate of convergence to efficiency obtained in a double auction context by Cripps and Swinkels (2006). The welfare analysis in the supply function model contrasts thus with the one in models where there is no endogenous public signal such as the Cournot market in Vives (1988), the beauty contest in Morris and Shin (2002), or the general linear-quadratic set up of Angeletos and Pavan (2007). With Cournot competition we have to add a deadweight loss due to private information. A large Cournot market does not aggregate information (i.e. a large Cournot market does not approach a full information competitive outcome) and in the limit there is a welfare loss due to private information.

A leading application of the model, as we will see in the next section, is to wholesale electricity markets. The model admits also other interpretations. The cost shock could be related to some ex post pollution or emissions damage which is assessed on the firm.

Before submitting its supply schedule a firm would receive some private information on this pollution damage. Still another interpretation of the shock would be a random opportunity cost of serving the market which is related to dynamic considerations. Revenue management deals with situations where the product, be it a hotel room, airline flight, generated electricity or tickets for a concert, has an expiration date and capacity is fixed well in advance and can be added only at high marginal cost.[4] The problem arises then of predicting the opportunity cost of sale (the value of a unit in a shortage situation). A high opportunity cost is an indication of high value of sales in the future. In this case a firm would have a private assessment of the opportunity cost with which it would place its supply schedule.

The plan of the paper is as follows. Section 2 introduces the application to electricity markets. Section 3 presents the supply function model with strategic firms and characterizes a linear Bayesian supply function equilibrium and its comparative static and welfare properties. Section 4 performs a welfare analysis (including a comparison with Bayesian Cournot equilibria). Section 5 characterizes the convergence to price-taking behavior as the market grows large. It provides also an analysis of the order of magnitude of deadweight losses. Concluding remarks, including potential policy implications, close the paper. All proofs and the analysis of the Bayesian Cournot model are gathered in the Appendix.

2. Application to electricity markets

A potential application of the model is to competition in the electricity spot market. In quite a few spot markets firms submit supply schedules in a day-ahead pool market which is organized as a uniform price multiunit auction. In the British Pool, the first liberalized wholesale market, generators had to submit a single supply schedule for the entire day. The schedules are increasing since the Pool's rules rank plants in order of increasing bids. Other wholesale markets have different rules (the British Pool was replaced by NETA in

[4] Talluri and Van Ryzin (2004) for the basics of revenue management.

2001).[5] In our modeling the supply functions are smooth (the old English pool was modeled like this by Green and Newbery (1992) and Green (1996, 1999)) while typically supplies are discrete. However the modeling of the auction with discrete supplies leads to existence problems of equilibrium in pure strategies (see von der Fehr and Harbord (1993)). The linear supply function model has been widely used in electricity markets and new developments include cost asymmetries, capacity constraints, piecewise affine supply functions and non-negativity generation constraints (see Baldick, Grant, and Kahn (2004) and Rudkevich (2005)).

Both strategic behavior and private information are relevant in electricity markets. There is ample evidence by now that firms bid over marginal costs (see, e.g. Borenstein and Bushnell (1999), Borenstein et al. (2002), Green and Newbery (1992), and Wolfram (1998)). Hortaçsu and Puller (2006) introduce private information on the contract positions of firms in the Texas balancing market (the day-ahead market is resolved with bilateral contracts).[6] Information on costs is available because the balancing market takes place very close to the generation moment and from information sellers. Kühn and Machado (2004) introduce private information on retail sales in their study of vertically integrated firms in the Spanish pool. Private cost information related to plant availability will be relevant when there is a day-ahead market organized as a pool where firms submit hourly or daily supply schedules.[7] Indeed, plant availability is random, and the firm has privileged information because of technical issues or transport problems; hydro availability in the reservoirs of each firm is private information; the terms of supply

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5 In the day-ahead market in the Spanish pool generators submit supply functions which have to be nondecreasing and can include up to 25 price-quantity pairs for each production unit, as well as some other ancillary conditions. The demand side can bid in a similar way and the market operator constructs a merit order dispatch by ordering in the natural way supply and demand bids. The intersection of the demand and supply schedules determines the (uniform) price. Once the market closes the system operator solves congestion problems and market participants may adjust their positions in a sequence of intra-day markets, which have similar clearing procedures as in the day-ahead market. (See Crampes and Fabra (2005).)

6 The authors also argue that to take a linear approximation to marginal costs in the Texas electricity market is reasonable.

7 Note that even if there was a market for information on costs the solution of the model with private information would yield the value of information.

contracts for energy inputs or imports are also private information. The latter include constraints in take-or-pay contracts for gas where the marginal cost of gas is zero until the constraint –typically private information to the firm– binds, or price of transmission rights in electricity imports depending on the private arrangements for the use of a congested interconnector. (It is worth noting that even if the opportunity costs of the inputs are the prices of those inputs in international markets in many instances there is not a single reference market.) Furthermore, in an emission rights system, future rights allocations may depend on current emissions and firms may have different private estimates of such allocation. This will affect the opportunity cost of using current emission rights.

There is a lively debate about the best way of modeling competition in the wholesale electricity market. The Cournot framework has been used in a variety of studies.[8] The advantage of the Cournot model is that it is a robust model in which capacity constraints and fringe suppliers are easily incorporated. A drawback is that the Cournot model tends to predict prices that are too high given realistic estimates of the demand elasticity.[9] The supply function approach is more realistic but potentially less robust. There is either non-existence of equilibrium in pure strategies if discrete supplies are taken into account or, as stated before, a plethora of equilibria in smooth models with no uncertainty. Baldick and Hogan (2006) justify to concentrate attention on linear supply function equilibria in a linear-quadratic model because other equilibria (in the range between the least competitive Cournot one and the most competitive) are unstable. A potential advantage of the supply function approach, over either the Cournot or the pure auction approaches, is that it implies that firms bid in a consistent way over an extended time horizon.

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8 See, for example, Borenstein and Bushnell (1999) for the US; Alba et al. (1999) and Ramos et. al. (1998) for Spain; and Andersson and Bergman (1995) for Scandinavia.

9 However, including vertical relations and contracts in a Cournot setting provides better estimates (see Bushnell, Mansur, and Saravia (2008)).

3. A strategic supply function model

Consider a market for a homogenous product with n consumers, each with quasilinear preferences and having the net benefit function

$$U(x) - px \text{ with } U(x) = \alpha x - \beta x^2/2,$$

where α and β are positive parameters and x the consumption level. This gives rise to the inverse demand $P_n(X) = \alpha - \beta X/n$ where X is total output. In the electricity market example the demand intercept α is a continuous function of time (load-duration characteristic) that yields the variation of demand over the time horizon considered. At any time there is a fixed α and the market clears.

There are n firms in the market also. We are considering thus an n-replica market and X/n is the average or per capita output. We will denote the average of a variable by a tilde (for example, $\tilde{x}_n = X/n$). Firm i produces according to a quadratic cost function

$$C(x_i;\theta_i) = \theta_i x_i + \frac{\lambda}{2} x_i^2$$

where θ_i is a random parameter and $\lambda > 0$. Total surplus is therefore given by $TS = nU(X/n) - \sum_i C(x_i, \theta_i)$ and per capita surplus by $TS/n = U(X/n) - \left(\sum_i C(x_i, \theta_i)\right)/n$.

As we will see below, this replica market can also be interpreted as a market parameterized by the number of consumers and where firms can enter freely paying a positive fixed entry cost. Then the free entry number of firms is of the order of the number of consumers. A large market is a market with a large number of consumers. We will consider in the paper the reduced-form replica market version instead of the free-entry version.

We assume that θ_i is normally distributed (with mean $\overline{\theta} > 0$ and variance σ_θ^2). The parameters θ_i and θ_j, $j \neq i$, are correlated with correlation coefficient $\rho \in [0,1]$. So we have $\mathrm{cov}[\theta_i, \theta_j] = \rho\sigma_\theta^2$, for $j \neq i$. Firm i receives a signal $s_i = \theta_i + \varepsilon_i$ and signals are of the same precision with ε_i normally distributed with $E[\varepsilon_i] = 0$ and $\mathrm{var}[\varepsilon_i] = \sigma_\varepsilon^2$. Error terms in the signals are uncorrelated among themselves and with the θ_i parameters. All random variables are thus normally distributed.

In the electricity example the random cost shock may be linked to plant availability because of technical issues or transport problems. Other shocks that may be private information to the firm are related to the level of hydro water in the reservoirs of the firm and the terms of the supply contracts for energy inputs or imports. Those terms are typically, at least partially, private information to the firm. This is so even if the opportunity costs of the contracted inputs are the prices of those inputs traded in international markets because there is not always a single reference market. The common component in the shock may be related to the prices of energy in international markets to which the supply contracts of firms are linked. Furthermore, the constraints in take-or-pay contracts for gas imply that the marginal cost of gas is zero until the constraint is hit. The point is that the level of the constraint is private information to the firm. For electricity imports, the price of transmission rights in electricity imports is also private information to the firm (for example, depending on the arrangements to use an interconnector subject to congestion). Finally, the internalization of costs of emission rights may depend on the private assessment of future rights allocations.

As stated in the introduction the cost shock could be also related to some ex post pollution or emission damage which is assessed on the firm and for which the firm has some private information. Another interpretation of the shock is a random opportunity cost of serving the market which is related to dynamic considerations (e.g. revenue management on the face of products with expiration date and costly capacity changes).

Ex-ante, before uncertainty is realized, all firms face the same prospects. It follows that the average parameter $\tilde{\theta}_n \equiv \left(\sum_{i=1}^{n}\theta_i\right)/n$ is normally distributed with mean $\overline{\theta}$, $\mathrm{var}\left[\tilde{\theta}_n\right] = \left(1+\left(n-1\right)\rho\right)\sigma_\theta^2/n$, and $\mathrm{cov}\left[\tilde{\theta}_n,\theta_i\right] = \mathrm{var}\left[\tilde{\theta}_n\right]$.

Our information structure encompasses the cases of "common value" and of "private values". For $\rho = 1$ the θ parameters are perfectly correlated and we are in a *common value* model. When signals are perfect, $\sigma_\varepsilon^2 = 0$ for all i, and $0 < \rho < 1$, we will say we are in a *private values* model. Agents receive idiosyncratic shocks, which are imperfectly correlated, and each agent observes his shock with no measurement error. When $\rho = 0$, the parameters are independent, and we are in an *independent values* model.

Let $\xi \equiv \sigma_\theta^2/\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)$, it is not difficult to see that

$$E\left[\theta_i \middle| s_i\right] = \xi s_i + \left(1-\xi\right)\overline{\theta} \text{ and } E\left[s_j \middle| s_i\right] = E\left[\theta_j \middle| s_i\right] = \xi\rho s_i + \left(1-\xi\rho\right)\overline{\theta}.$$

When signals are perfect, $\xi = 1$ and $E\left[\theta_i \middle| s_i\right] = s_i$, and $E\left[\theta_j \middle| s_i\right] = \rho s_i + \left(1-\rho\right)\overline{\theta}$. When they are not informative, $\xi = 0$ and $E\left[\theta_i \middle| s_i\right] = E\left[\theta_j \middle| s_i\right] = \overline{\theta}$.

Under the normality assumption conditional expectations are affine. There are other families of conjugate prior and likelihood that also yield affine conditional expectations and allow for bounded supports of the distributions. (See Vives (Ch. 2, 1999)).[10]

Firms compete in supply functions. We will restrict attention to symmetric Linear Bayesian Supply Function Equilibrium (LBSFE). The characterization of an equilibrium with supply function competition when there is market power and private information

[10] With normal distributions there is positive probability that prices and quantities are negative in equilibrium. This can be controlled by choice of the variances of the distributions and the parameters α, β, λ and $\overline{\theta}$.

needs to model the capacity of a firm to influence the market price at the same time that the firm learns from the price.

The strategy for firm i is a price contingent schedule $X(s_i,\cdot)$. This is a map from the signal space to the space of supply functions. Given the strategies of firms $X(s_j,\cdot)$, j = 1, …, n, for given realizations of signals market clearing implies that

$$p = P_n\left(\sum_{j=1}^{n} X(s_j,p)\right).$$

Let us assume that there is a unique market clearing price $\hat{p}(X(s_1,\cdot),...,X(s_n,\cdot))$ for any realizations of the signals.[11] Then profits for firm i, for any given realization of the signals, are given by

$$\pi_i(X(s_1,\cdot),...,X(s_n,\cdot)) = pX(s_i,p) - C(X(s_i,p))$$

where $p = \hat{p}(X(s_1,\cdot),...,X(s_n,\cdot))$. This defines a game in supply functions and we want to characterize a symmetric LBSFE. Let us posit a candidate symmetric equilibrium for the game with n firms:

$$X_n(s_i,p) = b_n - a_n s_i + c_n p .$$

Average output is given by $\tilde{x}_n = b_n - a_n\tilde{s}_n + c_n p$, where $\tilde{s}_n = \left(\sum_i s_i\right)/n = \tilde{\theta}_n + \left(\sum_i \varepsilon_i\right)/n$. Substituting in the inverse demand $p = \alpha - \beta\tilde{x}_n$ and solving for p we obtain

$$p = (1+\beta c_n)^{-1}(\alpha - \beta b_n + \beta a_n \tilde{s}_n),$$

where we posit that $1+\beta c_n > 0$.

Given the strategies of rivals $X_n(s_j,\cdot)$, j ≠ i, firm i faces a residual inverse demand

$$p = \alpha - \frac{\beta}{n}\sum_{j\neq i} X_n(s_j,p) - \frac{\beta}{n}x_i = \alpha - \frac{\beta}{n}(n-1)(b_n + c_n p_n) + \frac{\beta}{n}a_n\sum_{j\neq i} s_j - \frac{\beta}{n}x_i .$$

[11] If there is no market clearing price assume the market shuts down and if there are many then the one that maximizes volume is chosen.

It follows that

$$p = I_i - \frac{\beta}{n}\left(1+\beta\frac{n-1}{n}c_n\right)^{-1} x_i$$

where

$$I_i = \left(1+\beta\frac{n-1}{n}c_n\right)^{-1}\left(\alpha - \frac{\beta}{n}(n-1)b_n + \frac{\beta}{n}a_n\sum_{j\neq i}s_j\right).$$[12]

All the information provided by the price to firm i about the signals of others is subsumed in the intercept of residual demand I_i. The information available to firm i is therefore $\{s_i, p\}$ or, equivalently, $\{s_i, I_i\}$. Firm i chooses x_i to maximize

$$E\left[\pi_i \middle| s_i, p\right] = x_i\left(p - E\left[\theta_i \middle| s_i, p\right]\right) - \frac{\lambda}{2}x_i^2 = x_i\left(I_i - \frac{\beta}{n}\left(1+\beta\frac{n-1}{n}c_n\right)^{-1} x_i - E\left[\theta_i \middle| s_i, p\right]\right) - \frac{\lambda}{2}x_i^2.$$

The F.O.C. is

$$I_i - E\left[\theta_i \middle| s_i, I_i\right] - 2\frac{\beta}{n}\left(1+\beta\frac{n-1}{n}c_n\right)^{-1} x_i - \lambda x_i = 0$$

or, equivalently,

$$p - E\left[\theta_i \middle| s_i, p\right] = \left(\frac{\beta}{n+\beta(n-1)c_n} + \lambda\right)x_i.$$

The second order sufficient condition for a maximum is $\left(\frac{2\beta}{n+\beta(n-1)c_n} + \lambda\right) > 0$. An equilibrium must fulfill also $1+\beta c_n > 0$. The following proposition characterizes the linear equilibrium.

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12 Note that if $1+\beta c_n > 0$ then $1+\beta\frac{n-1}{n}c_n = 1+\beta c_n - \beta c_n / n > 0$ (either if $c_n > 0$ or $c_n < 0$).

Proposition 1. In the n-firm market with $\rho < 1$ and $\sigma_\varepsilon^2 / \sigma_\theta^2 < \infty$, there is a unique symmetric linear Bayesian supply function equilibrium. It is given by

$$X_n(s_i, p) = b_n - a_n s_i + c_n p,$$

where

$$a_n = \frac{(1-\rho)\sigma_\theta^2}{\left(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2\right)} \left(\frac{\beta}{n + \beta(n-1)c_n} + \lambda \right)^{-1}$$

$$b_n = \frac{1}{1+M_n} \left(\frac{\alpha}{\beta} M_n - \frac{\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2 + \left(1+(n-1)\rho\right)\sigma_\theta^2\right)} \left(\frac{\beta}{n + \beta(n-1)c_n} + \lambda \right)^{-1} \bar{\theta} \right)$$

and c_n is the largest solution to the quadratic equation

$$\begin{aligned} &\lambda\beta(n-1)(1+M_n)c_n^2 + \left((\beta+\lambda n)(1+M_n) + (\lambda M_n - \beta)(n-1)\right)c_n \\ &+ (\beta + \lambda n)\beta^{-1} M_n - n = 0 \end{aligned}$$

where $M_n \equiv \dfrac{\rho\sigma_\varepsilon^2 n}{(1-\rho)\left(\sigma_\varepsilon^2 + \left(1+(n-1)\rho\right)\sigma_\theta^2\right)}$. In equilibrium we have that $1 + \beta c_n > 0$, $a_n > 0$, and c_n decreases with M_n with $c_n > 0$ for $M_n = 0$.

Proof: See Appendix I.

The price p_n reveals the aggregate information $\tilde{s}_n$. The equilibrium is *privately revealing* (i.e. for firm i (s_i, p) or $(s_i, \tilde{s}_n)$ is a sufficient statistic of the joint information in the market, see Allen (1981)). The incentives to collect information are preserved because for firm i the signal s_i still helps in estimating θ_i even though p_n reveals $\tilde{s}_n$.

Some extreme cases

The equilibrium is in contrast with the pure common value model of Kyle (1989) where noise traders or noisy supply are needed in order to prevent the collapse of the market. In our model there is no noise and consequently in the pure common value case ($\rho = 1$

and $\sigma_\varepsilon^2 < \infty$) the market collapses. Indeed, when $\rho = 1$ and $\sigma_\varepsilon^2 < \infty$ a fully revealing REE is not implementable and there is no linear equilibrium. The reason should be well understood: if the price reveals the common value then no firm has an incentive to put any weight on its signal (and the incentives to acquire information disappear as well). But if firms put no weight on their signals then the price can not contain any information on the costs parameters. As $\rho \to 1$, $M_n \to \infty$ and at the linear equilibrium in Proposition 1 we have that $a_n \to 0$, $c_n \to -1/\beta$, $b_n \to \alpha/\beta$ and the equilibrium collapses in the limit. In fact, the supply function of a firm converges to the demand function $x = (\alpha - p)/\beta$.

Another particular case is when the signals are pure noise (i.e. $\sigma_\varepsilon^2 / \sigma_\theta^2 = \infty$) then there is always a linear equilibrium (even when $\rho = 1$). The equilibrium is given by

$$X_n(p) = c_n\left(p - \overline{\theta}\right)$$

where c_n is given implicitly by the *positive* root of

$$\left(\frac{\beta}{n + \beta(n-1)c_n} + \lambda\right)c_n = 1.$$

To see this note that if $\sigma_\varepsilon^2 / \sigma_\theta^2 = \infty$, then $E\left[\theta_i \middle| s_i, I_i\right] = \overline{\theta}$, $a_n = 0$ and

$$b_n = -\overline{\theta}\left(\frac{\beta}{n + \beta(n-1)c_n} + \lambda\right)^{-1} = -c_n\overline{\theta}.$$

When $\rho < 1$ and $\sigma_\varepsilon^2 / \sigma_\theta^2 \to \infty$ (in which case $M_n \to \rho n/(1-\rho)$) the equilibrium in Proposition 1 also collapses since $a_n \to 0$ (but there is a linear equilibrium in the limit as given above –which does not coincide with the limit of the equilibrium in Proposition 1 as $\sigma_\varepsilon^2 / \sigma_\theta^2 \to \infty$).

With private values (i.e. perfect signals with $\sigma_\varepsilon^2 = 0$) the price reveals $\tilde{\theta}_n$ and firm i already knows its cost θ_i. In this case $M_n = 0$ and the equilibrium is independent of ρ.

<u>Comparative statics</u>

The slope of supply c_n may be negative if costs shocks are correlated ($\rho > 0$) and signals not perfect ($\sigma_\varepsilon^2 > 0$). The price serves a dual role as index of scarcity and as conveyor of information. Indeed, a high price has a direct effect to increase the competitive supply of a firm, but also conveys news that costs are high. If $\rho = 0$ or $\sigma_\varepsilon^2 / \sigma_\theta^2 = 0$ then the price conveys no extra information on the costs of firm i and $c_n > 0$. As we have seen, this is the case also when there is no private information (i.e. signals are pure noise, $\sigma_\varepsilon^2 / \sigma_\theta^2 = \infty$).

As ρ or $\sigma_\varepsilon^2 / \sigma_\theta^2$ increase then the slope of the supply function becomes steeper (c_n decreases) because of the informational component of the price (i.e. the firm learns more from the price about its cost shock and reacts less to a price change than if the price was only an index of scarcity) and turns negative at some point. Indeed, it is easily checked that c_n decreases in ρ and $\sigma_\varepsilon^2 / \sigma_\theta^2$. This follows from the fact that the largest root of the quadratic equation determining c_n decreases with M_n and M_n is in turn increasing in ρ and $\sigma_\varepsilon^2 / \sigma_\theta^2$. Note that as σ_ε^2 increases the private signal of a firm diminishes its precision in a one-to-one fashion while the precision in the price diminishes according to the factor $1/(n-1)$. As ρ tends to 1, c_n becomes negative. There are particular parameter combinations (i.e. when $M_n = \left(n^{-1} + \lambda\beta^{-1}\right)^{-1}$) for which the scarcity and informational effects balance and firms set a zero weight ($c_n = 0$) on public information. In this case firms do not condition on the price and the model reduces to the Cournot model where firms compete in quantities. However, in this particular case, when supply functions are allowed, not reacting to the price (public information) is optimal.

The comparative statics results are reminiscent of asymmetric information models where traders submit steeper schedules to protect themselves against adverse selection (Kyle (1989), Biais, Martimort, Rochet (2000)). Kyle (1989) and Wang and Zender (2002) consider a common value model with noise. Biais et al. (2000) in a common value environment show that adverse selection reduces the aggressiveness of competition in supply schedules of risk neutral uninformed market makers facing a risk averse informed trader who is subject also to an endowment shock. The phenomenon is akin to the winner's curse in common value auctions (Milgrom and Weber (1982)): a bidder refrains from bidding aggressively because winning conveys the news that the signal the bidder has received was too optimistic (the highest signal in the pool). In our model a firm refrains from competing aggressively with its supply function because a high price conveys the bad new that costs are high. It is worth emphasizing that in the auction models of Kyle (1989) or Wang and Zender (2002) the *demand* schedules always have the "right" slope: downwards. Furthermore, in the double auction context of Kyle (1989) a linear equilibrium exists only if the number of informed traders is larger or equal than 3 (when there are no uninformed traders). This is so since the market breaks down when traders submit vertical schedules. In our model with strategic agents facing a demand function from passive consumers this does not happen.

<u>Applications</u>

In the pollution damage interpretation of the shock, and when the equilibrium calls for a downward sloping supply, we would have that when firms see the price going up they reduce supply because they figure out that the assessed pollution damage will be higher.

Patterns of pricing for airline flights have proved difficult to explain with extant theoretical models (see e.g. McAfee and te Velde (2006)). If we believe that supply function competition provides a suitable reduced form for pricing in such markets then taking into account the information aggregation role of price may help explaining some pricing patterns.[13] We have seen that when the information role of the price dominates is

[13] See Section 10.1 in Talluri and Van Ryzin (2004) for a description of airline pricing. For example, the authors state (p. 523): "A typical booking process proceeds as follows. An airline posts availability in

index of scarcity role then supply is downward sloping. If this is the case the interpretation would be that when airlines see prices going up they may infer, correctly, that the opportunity cost is high (i.e. that expected next period demand is high) and they reduce supply in the present period to be able to supply next period at a higher profit. In any case, when the variance of the (opportunity) cost shock increases the equilibrium reaction of firms is to set a flatter supply schedule inducing lower margins. Higher volatility of demand would translate then in lower margins.

Increasing the noise in the private signal σ_ε^2 makes the slope of supply steeper (decreases c_n). This result may help explain the fact that in the Texas balancing market small firms use steeper supply functions than those predicted by theory (Hortaçsu and Puller (2006)). Indeed, smaller firms may have signals of worse quality because of economies of scale in information gathering while private cost information has not been taken into account in the estimation.

<u>Competitiveness</u>

From the F.O.C. we have that

$$p-\left(E\left[\theta_i \left|s_i,p\right.\right]+\lambda x_i\right)=\left(\frac{1}{n\beta^{-1}+\left(n-1\right)c_n}\right)x_i,$$

where the slope of residual demand is $n\beta^{-1}+\left(n-1\right)c_n$. We see therefore that the competitiveness of the LBSFE depends on the slope of supply c_n. A consequence of the comparative statics results is that the margin over expected marginal cost $E\left[\theta_i \left|s_i,p\right.\right]+\lambda x_i$ is increasing in σ_ε^2 and ρ. Using simulations it can be checked that the aggregate slope of supply of rivals of a firm $\left(n-1\right)c_n$ increases (i.e. becomes flatter)

each fare class to the reservation systems stating the availability of seats in each fare class." This is not unlike a supply function.

with n whenever $c_n > 0$ for any n and that $n\beta^{-1} + (n-1)c_n$ is increasing in n always. It follows that the margin is decreasing in n. [14]

A similar relation holds for the margin over average expected marginal cost

$$E_n[MC_n] \equiv \frac{1}{n}\sum_{i=1}^{n}\left(E\left[\theta_i \middle| s_i, p\right] + \lambda x_i\right) = \frac{1}{n}\sum_{i=1}^{n} E\left[\theta_i \middle| s_i, p\right] + \lambda \tilde{x}_n :$$

$$\frac{p - E_n[MC_n]}{p} = \frac{1}{\left(n + \beta(n-1)c_n\right)\eta_n}$$

where $\eta_n = p/\left(\beta\tilde{x}_n\right)$ is the elasticity of demand.

A Bayesian Cournot equilibrium, where firm i sets a quantity contingent only on its information $\{s_i\}$, corresponds to $c_n = 0$ with a margin $1/n\eta_n$ (the unique Bayesian Cournot equilibrium is derived formally in Proposition A.1 in Appendix II). The supply function and the Cournot equilibrium coincide when $M_n = \left(n^{-1} + \lambda\beta^{-1}\right)^{-1}$, in which case $c_n = 0$ (indeed, then it can be checked that $a_n^{\mathrm{Cournot}} = a_n^{\mathrm{SF}}$ and $b_n^{\mathrm{Cournot}} = b_n^{\mathrm{SF}}$).

When $c_n > 0$ we are in the usual case in which the supply function equilibrium has positive slope and is between the Cournot and the competitive outcomes (e. g. Klemperer and Meyer (1989) when uncertainty has unbounded support). However, when $c_n < 0$ the margin is larger than the Cournot level and, in fact, converges to the collusive level $1/\eta_n$ when $\rho \to 1$ (this is so since $\left(n + \beta(n-1)c_n\right) \to 1$ as $\rho \to 1$). It is remarkable that firms

[14] Another possible pattern is for $(n-1)c_n$ to have a hump-shaped form with n being positive and increasing first to become decreasing and eventually negative. (c_n may increase or decrease with n.) Simulations have been performed for the range of parameters σ_ε^2 and σ_θ^2 in $\{0.1, 1, 10\}$ and β and $\lambda \in \{1, 10\}$, setting α and $\bar{\theta}$ so as to control the probability that negative prices and/or quantities occur (say, letting, $\alpha = 10$ and $\bar{\theta} = 5$ or $\alpha = 30$ and $\bar{\theta} = 15$).

may approach collusive margins in a one-shot noncooperative equilibrium because of informational reasons.

In some cases the rules of the market force supply functions to be nondecreasing. If this is the case and if the equilibrium unrestricted supply were to be downward sloping then the restricted equilibrium would be of the Cournot type with firms submitting vertical schedules. This implies that the market power of firms is capped at the Cournot level.

The following proposition summarizes results so far.

Proposition 2. At the LBSFE, with $\rho < 1$ and $\sigma_\varepsilon^2 / \sigma_\theta^2 < \infty$, the slope of equilibrium supply is steeper (c_n smaller) with increases in ρ and $\sigma_\varepsilon^2 / \sigma_\theta^2$, going from $c_n > 0$ for $\rho = 0$ or $\sigma_\varepsilon^2 = 0$ to $c_n < 0$ for large values of ρ or $\sigma_\varepsilon^2 / \sigma_\theta^2$. As $\rho \to 1$ the margin over average expected marginal cost tends to the collusive level.

Free entry

The replica market considered can be the outcome of free entry in a market parameterized by size. Consider a market with m consumers (the size of the market) and inverse demand $P_m(X) = \alpha - \beta_m X$ where $\beta_m = \beta / m$. Suppose now that at a first stage firms decide whether to enter the market or not. If a firm decides to enter it pays a fixed cost $F > 0$. At a second stage each active firm i, upon observing its signal s_i, sets an output level. Given that n firms have entered, a Bayesian Supply Function equilibrium is realized. Given our assumptions, for any n there is a unique, and symmetric, linear equilibrium yielding expected profits $E[\pi_n] = ((\lambda n + 2\beta)/2n) E\left[(X_n(s_i, p))^2\right]$ for each firm. A free entry equilibrium is a subgame-perfect equilibrium of the two-stage game. A subgame-perfect equilibrium requires that for any entry decisions at the first stage, a Bayesian-Nash equilibrium in supply functions obtains at the second stage. Given a market of size m, the free entry number of firms $n^*(m)$ is approximated by the solution to $E[\pi_n] = F$ (provided F is not so large to prevent any entry). It can be checked that $n^*(m)$ is of the same order as m (similarly as in Vives (2002)). This means that the ratio of consumers to firms is bounded away from zero and infinity for any market size. We can reinterpret, therefore, the replica market as a free entry market parameterized by market size.

4. Welfare analysis

In order to assess the welfare loss due to strategic behavior we characterize price-taking equilibria. We complement the analysis comparing with the performance of Bayesian Cournot equilibria.

4.1 Price-taking equilibria and deadweight losses

Full (shared) information competitive equilibria are Pareto optimal and characterized by the equality of price and expected marginal cost (with full information):

$$p = E\left[\theta_i \left| s_i, \tilde{s}_n \right.\right] + \lambda x_i,\ \ i = 1, .., n.$$

The implied allocation is symmetric (since the total surplus optimization problem is strictly concave and firms and information structure are symmetric) and, provided that $\rho < 1$, $\sigma_\theta^2 > 0$ and $\sigma_\varepsilon^2 < \infty$, it is implemented by a symmetric price-taking LBSFE (denoted by a superscript "c" –for competitive– on the coefficients). The equilibrium strategy will be of the form $X_n^c\left(s_i, p\right) = b_n^c - a_n^c s_i + c_n^c p$ and it will arise out of the maximization of expected profits of firm i taking prices as given but using the information contained in the price. That is, firm i chooses x_i to maximize

$$E\left[\pi_i \left| s_i, p \right.\right] = x_i\left(p - E\left[\theta_i \left| s_i, p \right.\right]\right) - \frac{\lambda}{2} x_i^2.$$

This will yield the following system of F.O.C.

$$p = E\left[\theta_i \left| s_i, p \right.\right] + \lambda x_i \text{ for i = 1, .., n,}$$

where $p=\left(1+\beta c_n^c\right)^{-1}\left(\alpha-\beta b_n^c+\beta a_n^c\tilde{s}_n\right)$ provided that $1+\beta c_n^c \neq 0$. In the linear equilibrium we obtain $1+\beta c_n^c \neq 0$ > 0 and $a_n^c > 0$. Therefore, p reveals $\tilde{s}_n$, $E\left[\theta_i \middle| s_i, p\right]=E\left[\theta_i \middle| s_i, \tilde{s}_n\right]$, and the price-taking LBSFE implements the efficient solution.

Indeed, similarly as in the proof of Proposition 1 it can be checked that the coefficients of the equilibrium strategy $X_n^c\left(s_i, p\right)=b_n^c-a_n^c s_i+c_n^c p$ are given by the solution to the system of equations

$$\begin{cases} \dfrac{(1-\rho)\sigma_\theta^2}{\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)}=\lambda a_n \\ -\dfrac{\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2+\left(1+(n-1)\rho\right)\sigma_\theta^2\right)}\bar{\theta}-\dfrac{M_n\left(\beta b_n-\alpha\right)}{\beta}\lambda=\lambda b_n \\ \left(1-\dfrac{M_n\left(1+\beta c_n\right)}{\beta}\lambda\right)=\lambda c_n \end{cases}$$

where M_n is as in Section 3.

The following proposition is immediate.

<u>Proposition 3.</u> In the n-firm market with ρ < 1 and $\sigma_\varepsilon^2/\sigma_\theta^2<\infty$, there is a unique symmetric price-taking LBSFE. It is given by $X_n^c\left(s_i, p\right)=b_n^c-a_n^c s_i+c_n^c p$ with

$$a_n^c=\frac{(1-\rho)\sigma_\theta^2}{\lambda\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)}, b_n^c=\frac{1}{1+M_n}\left(\frac{\alpha}{\beta}M_n-\frac{\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2+\left(1+(n-1)\rho\right)\sigma_\theta^2\right)}\lambda^{-1}\bar{\theta}\right),$$

$$\text{and } c_n^c=\frac{\lambda^{-1}-\beta^{-1}M_n}{M_n+1}, \text{and } M_n \equiv \frac{\rho\sigma_\varepsilon^2 n}{(1-\rho)\left(\sigma_\varepsilon^2+\left(1+(n-1)\rho\right)\sigma_\theta^2\right)}.$$

This equilibrium implements the efficient allocation.

Note that $1+\beta c_n^c>$ 0 and $a_n^c>0$ and that we may have also c_n^c < 0. As before the equilibrium supply function can be upward or downward sloping. It will be downward sloping when the reaction to private information is small (i.e. when we are close to the common value case, when prior uncertainty is low or noise in the signals is high).

We have that c_n^c strictly decreases with n provided ρ > 0 (the result follows since c_n^c is strictly decreasing in M_n and M_n can be shown to be strictly increasing in n if ρ > 0). The reason that the supply function at a price-taking LBSFE is steeper as n grows is that with larger n the price has better information about costs (and this matters if ρ > 0). When ρ = 0 we have that $M_n = 0$, $c_n^c = 1/\lambda$, and $b_n^c = -a_n^c\overline{\theta}$ (in this case, obviously, $x_i = \lambda^{-1}\left(p - E\left[\theta_i \middle| s_i\right]\right)$).

For ρ < 1, and $\sigma_\varepsilon^2 / \sigma_\theta^2 < \infty$, the supply function of a firm in the price-taking equilibrium is always flatter than the supply function of the firm in the strategic equilibrium:

$$c_n^c - c_n = \left(\lambda^{-1} - \left(\frac{\beta}{n + \beta(n-1)c_n} + \lambda\right)^{-1}\right) / \left(M_n + 1\right) > 0$$

since in equilibrium $n + \beta(n-1)c_n > 0$. Similarly, we obtain that $b_n^c - b_n > 0$ provided that $\sigma_\varepsilon^2 > 0$. It is immediate also that firms are more cautious responding to their private signals when they have market power:

$$a_n^c - a_n = \frac{(1-\rho)\sigma_\theta^2}{\left(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2\right)}\left(\lambda^{-1} - \left(\frac{\beta}{n + \beta(n-1)c_n} + \lambda\right)^{-1}\right) > 0 .$$

This is because of the usual effect of market power: A firm takes into account the price impact coming from his production. Note that in principle a firm with market power would also be cautious because of the informational leakage from his action, but here the equilibrium is revealing. The simulations also uncover that price volatility is always larger in the competitive case.

The (expected) deadweight loss at the LBSFE is the difference between (per capita) expected total surplus at the LBSFE (ETS_n) and at the price-taking LBSFE (ETS_n^c), $\left(ETS_n^c - ETS_n\right)/n$. It can be shown (see Appendix I) that

$$\left(ETS_n^c - ETS_n\right)/n = \left(\beta E\left[\left(\tilde{x}_n - \tilde{x}_n^c\right)^2\right] + \lambda E\left[\left(x_{in} - x_{in}^c\right)^2\right]\right)/2.$$

The deadweight loss is due to market power. Simulations show that a typical case is for the deadweight loss to be an increasing function of ρ and of σ_ε^2 (with ETS_n decreasing and ETS_n^c U-shaped in ρ, and with both ETS_n and ETS_n^c decreasing in σ_ε^2).[15] This is true both for the part of the deadweight loss corresponding to allocative inefficiency as well as the one corresponding to productive inefficiency (see the decomposition in Appendix I). This is consistent with the result that as $\rho \to 1$ and we approach the common value case margins tend to the collusive level. It also found that the deadweight loss decreases as the market gets large (i.e. with n). (See Figure 1a, b).

However, other patterns are possible and the deadweight loss may be decreasing in ρ. This happens, for example, when $\beta = 4, \lambda = 0.5$, $\overline{\theta} = 20$, $\alpha = 45$, and $\sigma_\varepsilon^2 = 0.2, \sigma_\theta^2 = 10$. In this case ETS_n^c falls with ρ more quickly than ETS_n.

As we increase ρ the distance between the LBSFE and its price-taking counterpart may increase or decrease. There are several effects to consider. First, increasing ρ decreases c_n (i.e. makes the LBSFE less competitive) and this tends to increase both $a_n^c - a_n$ and $c_n^c - c_n$ for given value of ρ. Second, the very increase of ρ, for a given c_n, reduces $a_n^c - a_n$ and $c_n^c - c_n$ (the latter since M_n is monotone increasing in ρ). The second effect is due to the fact that increases in ρ make reliance on private information less important. Finally, there is another effect on expected total surplus via the variability of the average cost parameter $\mathrm{var}\left[\tilde{\theta}_n\right]$. This third effect has nothing to do with information and is highlighted in the private values case ($\sigma_\varepsilon^2 = 0$) where both competitive and strategic

15 The parameters considered systematically for the simulation are $\alpha = 10$ and $\overline{\theta} = 5$ or $\alpha = 30$ and $\overline{\theta} = 15$, and $\beta, \lambda \in \{1, 10\}$, and $\sigma_\theta^2, \sigma_\varepsilon^2 \in \{0.1, 1, 10\}$.

equilibria are independent of ρ. In this case increasing ρ decreases the deadweight loss since ETS_n^c falls faster with ρ than ETS_n by effect of the induced increase in $\mathrm{var}\left[\tilde{\theta}_n\right]$.[16]

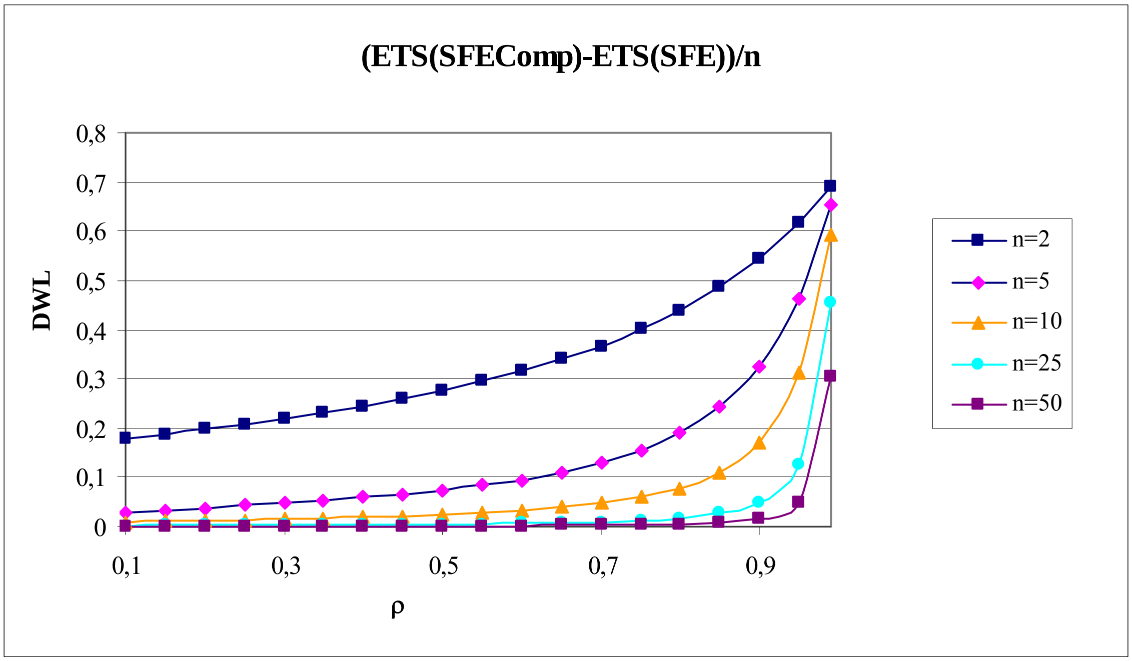

Figure 1a. Deadweight loss as a function of ρ (with parameters $\beta = \lambda = 1$, $\overline{\theta} = 5$, $\alpha = 10$, and $\sigma_\varepsilon^2 = \sigma_\theta^2 = 1$).

16 The likely reason is that increasing ρ increases $\mathrm{var}\left[\tilde{\theta}_n\right]$ and this increases the variance of average output, which in turn decreases expected total surplus (expected consumer surplus increases, and expected profits decrease, with higher average output variability since then consumers get lower prices when they consume more; the profit decrease dominates the consumer surplus increase). The differential effect on the competitive surplus is due to the fact that the competitive output is more sensitive to costs and therefore the impact on output variability of the increase in ρ larger.

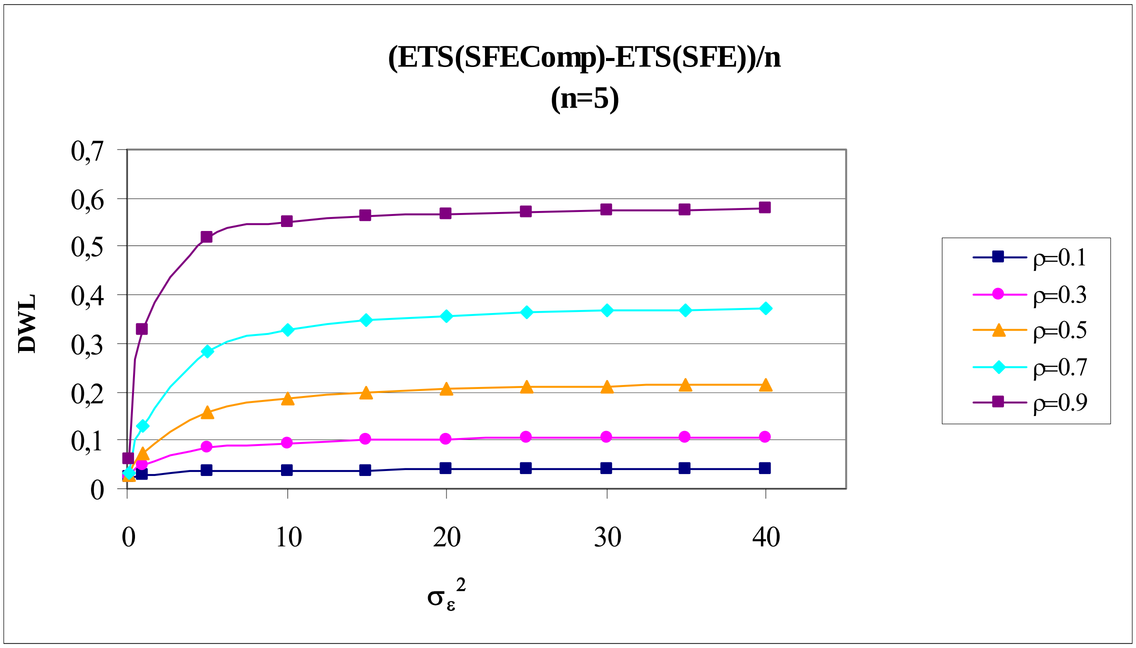

Figure 1b. Deadweight loss as a function of σ_ε^2 (with parameters n = 5, $\beta=\lambda=1, \overline{\theta}=5, \alpha=10$, and $\sigma_\theta^2=1$).

4.2 Comparison with Cournot

The welfare evaluation of the LBSFE is in marked contrast with the Bayesian Cournot equilibrium. In contrast to the Cournot case, at the LBSFE there is only a deadweight loss due to market power but not due to private information. There is always a welfare loss at the price-taking Bayesian Cournot equilibrium because the Cournot market mechanism does not aggregate information. However, a price-taking Bayesian Cournot equilibrium is team optimal (i.e. maximizes total expected surplus subject to the constraint that firms use decentralized -quantity- strategies in information, see Vives (1988)).

It is worth to compare the relative efficiency of the Cournot market ($ETS_n^{Cournot}$) in relation to the supply function market (ETS_n^{SF}) in per capita terms. A typical pattern for moderate n is for $\left(ETS_n^{SF}-ETS_n^{Cournot}\right)/n$ to be positive for ρ close to zero and negative for ρ close to 1, being zero at the point for which the supply function equilibrium calls for a vertical supply. For larger n we may have $\left(ETS_n^{SF}-ETS_n^{Cournot}\right)/n>0$ all along. (See

Figure 2.) In fact, for a given ρ and for large n we have always that $\left(ETS_n^{SF} - ETS_n^{Cournot}\right)/n > 0$. (See Figure 3.) Furthermore, when signals are perfect ($\sigma_\varepsilon^2 = 0$) we have also that $\left(ETS_n^{SF} - ETS_n^{Cournot}\right)/n > 0$ always. When signals are close to perfect we have that $\left(ETS_n^{SF} - ETS_n^{Cournot}\right)/n > 0$ except for ρ very close to 1.

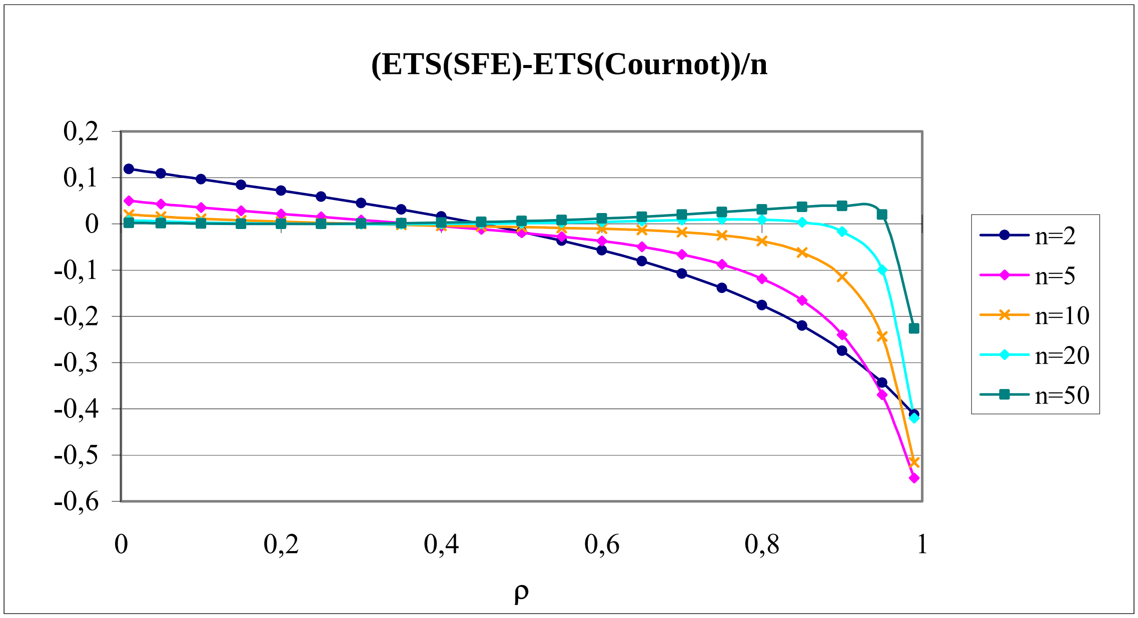

Figure 2. Efficiency differential between supply function and Cournot equilibria as a function of ρ (with parameters $\beta = \lambda = 1$, $\overline{\theta} = 5$, $\alpha = 10$, and $\sigma_\varepsilon^2 = \sigma_\theta^2 = 1$).

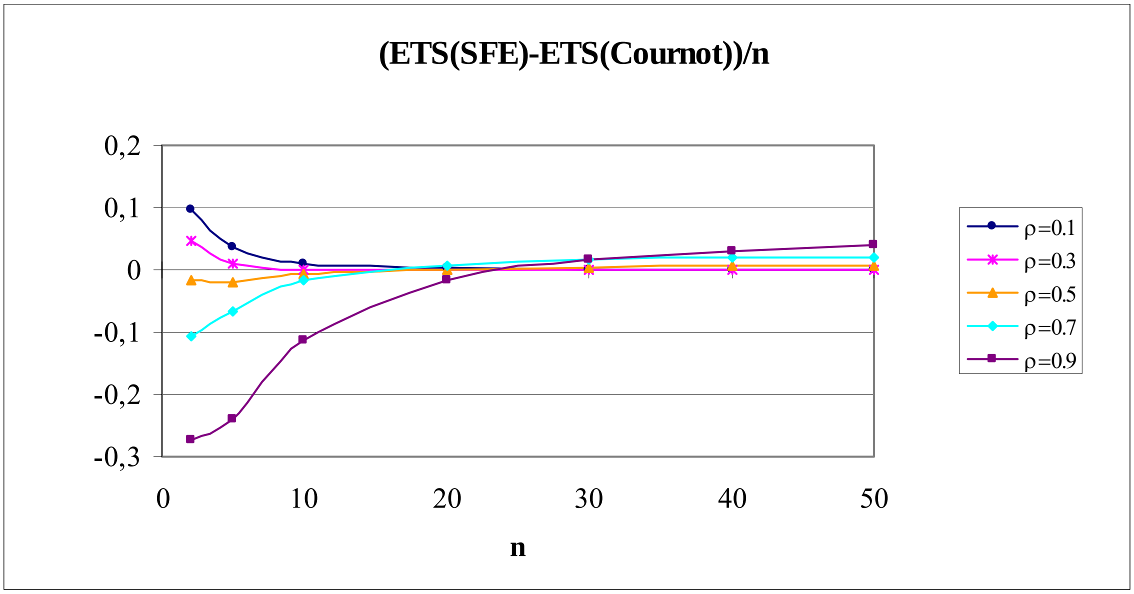

Figure 3. Efficiency differential between supply function and Cournot equilibria as a function of n (with parameters $\beta = \lambda = 1, \bar{\theta} = 5, \alpha = 10$, and $\sigma_\varepsilon^2 = \sigma_\theta^2 = 1$).

The intuition for the results should be clear. The supply function market always dominates in efficiency in terms of information because firms have full information while in the Cournot market they do not. For $\sigma_\varepsilon^2 = 0$ or ρ small the supply function market dominates overall because on top firms have less market power (since supply functions slope upwards). The result is that $ETS_n^{SF} - ETS_n^{Cournot} > 0$ for $\sigma_\varepsilon^2 = 0$ or ρ small. For larger ρ and $\sigma_\varepsilon^2 > 0$, when supply functions slope downwards, firms in the supply function market have more market power and this may dominate the information effect for n low, with the result that $ETS_n^{SF} - ETS_n^{Cournot} < 0$. At the critical value of ρ for which the supply function equilibrium calls for a vertical supply we have $ETS_n^{SF} - ETS_n^{Cournot} = 0$ since then both equilibria coincide. For n larger the market power effect is not very important and the information effect dominates and we have that $\left(ETS_n^{SF} - ETS_n^{Cournot}\right)/n > 0$ always.

For n large, for a fixed ρ, we must have $\left(ETS_n^{SF} - ETS_n^{Cournot}\right)/n > 0$ since as n grows the supply function equilibrium converges to the (full information) first best but not the

Cournot equilibrium. At the LBSFE there is only a deadweight loss due to market power, which dissipates in a large market; at the Bayesian Cournot equilibrium the deadweight loss due to private information remains in a large market (see Section 5 for the formal results).

The comparison between the Cournot and supply function outcomes has practical implications. First when a supply function market is modeled, for convenience, à la Cournot we want to know what biases are introduced. Second, when firms are restricted to use upward sloping schedules they may end up using vertical ones when in the supply function equilibrium they would be called to use downward sloping ones. The restriction to upward sloping schedules caps the market power of firms in the supply function market.

With respect to the first issue, recall that at the LBSFE there is only a deadweight loss due to market power but not due to private information. The result is that in a market characterized by supply function competition using the Cournot model overestimates the welfare loss with respect to the actual supply function mechanism on two counts when supply function slope upwards: excessive market power and lack of information aggregation. When the equilibrium supply function slopes downwards the Cournot market underestimates market power and then the comparison is ambiguous: the Cournot market may under- or overestimate the deadweight loss in relation to supply function competition. In a large market, as we will see in the next section, the Cournot model always overestimates the welfare loss since at the LBSFE there is (almost) no efficiency loss while there is a significant one with Cournot competition due to private information.

With respect to the second issue, forcing firms to use increasing schedules when they would like to use decreasing ones may or not may be a good idea depending basically on the number of firms. It will be good when the number of firms (and size of the market) is moderate.

5. Convergence properties

We show next how equilibria in finite economies become price-taking. We characterize the rates of convergence and the order of magnitude of the deadweight loss at the LBSFE.

Before stating the convergence results we will recall some measures of speed of convergence. We say that the sequence (of real numbers) b_n is of the *order* n^{υ}, with υ a real number,. whenever $n^{-\upsilon} b_n \xrightarrow[n]{} k$ for some nonzero constant k. We say that the sequence of random variables $\{y_n\}$ converges in *mean square* to zero at the rate $1/\sqrt{n^r}$ (or that y_n is of the order $1/\sqrt{n^r}$) if $E\left[(y_n)^2\right]$converges to zero at the rate $1/n^r$ (i.e. $E\left[(y_n)^2\right]$ is of the order $1/n^r$). Given that $E\left[(y_n)^2\right] = \left(E[y_n]\right)^2 + \mathrm{var}[y_n]$, a sequence $\{y_n\}$ such that $E[y_n] = 0$ and $\mathrm{var}[y_n]$ is of the order of $1/n$, converges to zero at the rate $1/\sqrt{n}$.

If the random parameters $(\theta_1, ..., \theta_n)$ are i.i.d. with finite variance and mean $\bar{\theta}$, and we let $\tilde{\theta}_n \equiv \left(\sum_{i=1}^{n} \theta_i\right)/n$, then $\tilde{\theta}_n - \bar{\theta}$ converges (in mean square) to 0 at the rate of $1/\sqrt{n}$ because $E\left[\tilde{\theta}_n - \bar{\theta}\right] = 0$ and $\mathrm{var}\left[\tilde{\theta}_n\right] = \sigma_\theta^2 / n$. In our case $\tilde{\theta}_n$ is normally distributed with mean $\bar{\theta}$ and $\mathrm{var}\left[\tilde{\theta}_n\right] = \left(1 + (n-1)\rho\right)\sigma_\theta^2 / n$. We have therefore that $\tilde{\theta}_n \to \tilde{\theta}$ in mean square at the rate $1/\sqrt{n}$ where $\tilde{\theta}$ is normally distributed with mean $\bar{\theta}$ and variance $\rho\sigma_\theta^2$.

The following proposition characterizes the convergence of the LBSFE to a price-taking equilibrium as the market grows. As we have seen before the price-taking equilibrium is first best efficient since it aggregates information.

Proposition 4. As the market grows large the market price p_n (at the LBSFE) converges in mean square to the price-taking Bayesian price p_n^c at the rate of $1/n$. (That is,

$E\left[\left(p_n - p_n^c\right)^2\right]$ tends to 0 at the rate of $1/n^2$.) The deadweight loss at the LBSFE $\left(ETS_n^c - ETS_n\right)/n$ is of the order of $1/n^2$.

The results follow because $a_n^c - a_n$, $b_n^c - b_n$, and $c_n^c - c_n$ are at most of the order of $1/n$ and both p_n and p_n^c depend on the average signal $\tilde{s}_n$. (See Appendix I.)

The rate of convergence to price taking behavior is 1/n, which is the same as the usual rate under complete information. The departure from price taking (marginal cost) is of the order of 1/n and the deadweight loss is of the order of the square of it. This result should not be surprising since the LBSFE aggregates information. Cripps and Swinkels (2006) obtain a parallel result in a double auction environment. The authors consider a generalized private value setting where bidders can be asymmetric and can demand or supply multiple units. Under some regularity conditions (and a weak requirement of "a little independence" where each player's valuation has a small idiosyncratic component), they find that as the number of players n grows (say that there are n buyers and n sellers) all nontrivial equilibria of the double auction converge to the competitive outcome and inefficiency vanishes at the rate of $1/n^{2-\alpha}$ for any $\alpha > 0$.

It follows from the simulations also that typically the speed of convergence of the deadweight loss to zero (in terms of the constant of convergence) is slower when ρ is larger. That is, the limit as n tends to infinity of $n\left(ETS_n^c - ETS_n\right)$ is increasing with ρ. (This is so since we have seen that $\left(ETS_n^c - ETS_n\right)$ is typically increasing in ρ for any n and the limit of $n\left(ETS_n^c - ETS_n\right)$ is well defined .)

At the Bayesian Cournot equilibrium Proposition 4 holds (with the important proviso that now $\left(ETS_n^c - ETS_n\right)/n$ is the deadweight loss with respect to the price-taking equilibrium but not with respect to the full information first best). The price-taking Bayesian Cournot equilibrium coincides with full information first best in a large economy only in the independent values case, where there is no aggregate uncertainty (see Vives (2002)). Otherwise, as the market grows large there is no convergence to a full information equilibrium.

In summary, convergence to price-taking is, in both cases, at the rate of $1/n$ for prices and $1/n^2$ for the welfare loss with respect to price-taking behavior.

6. Concluding remarks

In this paper we have examined supply function competition with private information and compared it with the Cournot model. While in some markets the supply function model is closer to the institutional set up it is often the case that the Cournot model is used instead. The reason is that the Cournot model is easier to handle and quite robust. The question arises then on the nature of the biases introduced by the Cournot model in this situation. In our linear-normal model we have found that there is a unique symmetric linear Bayesian supply function equilibrium. The equilibrium is privately revealing with the private signal of the firm and the price being sufficient statistics of the join information in the market for the firm. This means that the incentives to acquire information are preserved.

Supply functions may slope downwards when the information role of the price overwhelms its traditional index of scarcity role. This may help explain odd pricing patterns. Furthermore, we find that an increase in the correlation of cost parameters or in the noise in private signals makes supply functions steeper when upward sloping and increases price-cost margins always. This implies that ignoring private cost information with supply function competition may explain supply slopes that look “too high”. For example, in the Texas balancing market small firms may use steeper supply functions than those predicted by theory (Hortaçsu and Puller (2006)) since small firms have signals of worse quality because of economies of scale in information gathering.

The result may have regulatory and competition policy implications since the observation of high margins may be taken as an indication of excessive market power or collusion (coordinated behavior) when in fact it may be explained by poor information of the part of firms. Indeed, close to the common value situation margins will approach the collusive level and the deadweight loss will increase. However, from the competition policy perspective it is useless in this market to go after coordinated behavior since firms are just adjusting noncooperatively to private information. It may be useful instead to cap the market power of firms (e.g. requiring them to use upward sloping schedules). This will improve welfare in markets of moderate size and number of firms.

The welfare analysis provides a stark contrast between Cournot and supply function equilibria. Indeed, the Cournot model overestimates the welfare loss with respect to an actual supply function mechanism on two counts when supply functions are upward sloping: excessive market power and lack of information aggregation. The Cournot model displays too high margins and an increased welfare loss since firms only rely on their private information. When supply functions slope downwards then Cournot underestimates the market power of firms and the comparison is ambiguous: the Cournot market may under- or overestimate the deadweight loss in relation to supply function competition. In both the supply function and Cournot models the order of magnitude of the distortion because of strategic behavior is 1/n in prices and $1/n^2$ in the deadweight loss where n is the number of firms (and size of the market). This result provides a measure of the competitiveness of the market. The difference is that in a large market at the supply function equilibrium there is no efficiency loss while there is with Cournot competition due to private information.

Several extensions may be worth pursuing. Among them, the consideration of asymmetric firms (and merger analysis), the introduction of forward contracts, and costly information acquisition.

Appendix I: Proofs of Propositions 1, 2, 3 and 4.

Proof of Proposition 1: Consider a candidate linear equilibrium $X_n(s_i,p)=b_n-a_ns_i+c_np$. Positing $1+\beta c_n>0$, the price equation

$$p=(1+\beta c_n)^{-1}(\alpha-\beta b_n+\beta a_n\tilde{s}_n)$$

provided that $a_n>0$ can be rearranged to define

$$h_i\equiv\frac{p(1+\beta c_n)-\alpha+\beta b_n}{\beta a_n}n-s_i=\sum_{j\neq i}s_j.$$

The pair (s_i,p) is informationally equivalent to the pair (s_i,h_i), hence

$$E\left[\theta_i\middle|s_i,p\right]=E\left[\theta_i\middle|s_i,h_i\right].$$

Because of the assumed information structure we have

$$\begin{pmatrix}\theta_i\\ s_i\\ h_i\end{pmatrix}\sim N\left(\begin{pmatrix}\bar{\theta}\\ \bar{\theta}\\ (n-1)\bar{\theta}\end{pmatrix},\begin{pmatrix}\sigma_\theta^2 & \sigma_\theta^2 & (n-1)\rho\sigma_\theta^2\\ \sigma_\theta^2 & \sigma_\theta^2+\sigma_\varepsilon^2 & (n-1)\rho\sigma_\theta^2\\ (n-1)\rho\sigma_\theta^2 & (n-1)\rho\sigma_\theta^2 & \psi\end{pmatrix}\right),$$

where $\psi=(n-1)(\sigma_\theta^2+\sigma_\varepsilon^2)+(n-1)(n-2)\rho\sigma_\theta^2$. We obtain

$$E\left[\theta_i\middle|s_i,h_i\right]=E\left[\theta_i\middle|s_i,\frac{h_i}{n-1}\right]=\frac{\sigma_\varepsilon^2}{\sigma_\theta^2(1+(n-1)\rho)+\sigma_\varepsilon^2}\bar{\theta}+$$

$$\frac{\sigma_\theta^2\left(\sigma_\theta^2(1-\rho)\left(1+(n-1)\rho\right)+\sigma_\varepsilon^2\right)}{\left(\sigma_\theta^2(1-\rho)+\sigma_\varepsilon^2\right)\quad\left(\sigma_\theta^2\left(1+(n-1)\rho\right)+\sigma_\varepsilon^2\right)}s_i+\frac{\sigma_\theta^2\sigma_\varepsilon^2\rho}{\left(\sigma_\theta^2(1-\rho)+\sigma_\varepsilon^2\right)\ \left(\sigma_\theta^2\left(1+(n-1)\rho\right)+\sigma_\varepsilon^2\right)}h_i.$$

We are looking for strategies of the form $X_n(s_i,p)=b_n-a_ns_i+c_np$. Using the F.O.C. $p-E\left[\theta_i\middle|s_i,p\right]=\left(\frac{\beta}{n+\beta(n-1)c_n}+\lambda\right)x_i$ and the expression for h_i we obtain the following

$$-\frac{\sigma_\varepsilon^2\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)\bar{\theta}+\left(\sigma_\theta^2\sigma_\varepsilon^2 n\rho\left(\beta b_n-\alpha\right)/\beta a_n\right)}{\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)\left(\sigma_\varepsilon^2+(1+(n-1)\rho)\sigma_\theta^2\right)}-\frac{(1-\rho)\sigma_\theta^2\left(\sigma_\varepsilon^2+(1+(n-1)\rho)\sigma_\theta^2\right)}{\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)\left(\sigma_\varepsilon^2+(1+(n-1)\rho)\sigma_\theta^2\right)}s_i$$

$$+\left(1-\frac{\sigma_\theta^2\sigma_\varepsilon^2 n\rho\left((1+\beta c_n)/\beta a_n\right)}{\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)\left(\sigma_\varepsilon^2+(1+(n-1)\rho)\sigma_\theta^2\right)}\right)p=\left(\frac{\beta}{n+\beta(n-1)c_n}+\lambda\right)\left(b_n-a_n s_i+c_n p\right)$$

Identifying coefficients, letting $M_n \equiv \frac{\rho\sigma_\varepsilon^2 n}{(1-\rho)\left(\sigma_\varepsilon^2+(1+(n-1)\rho)\sigma_\theta^2\right)}$, we obtain a_n, b_n, c_n by solving the following system of equations

$$\begin{cases} \dfrac{(1-\rho)\sigma_\theta^2}{\left(\sigma_\varepsilon^2+(1-\rho)\sigma_\theta^2\right)}=\left(\dfrac{\beta}{n+\beta(n-1)c_n}+\lambda\right)a_n \\ -\dfrac{\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2+(1+(n-1)\rho)\sigma_\theta^2\right)}\bar{\theta}\left(\dfrac{\beta}{n+\beta(n-1)c_n}+\lambda\right)^{-1}-\dfrac{M_n\left(\beta b_n-\alpha\right)}{\beta}=b_n \\ \left(1-\dfrac{M_n\left(1+\beta c_n\right)}{\beta}\left(\dfrac{\beta}{n+\beta(n-1)c_n}+\lambda\right)\right)=\left(\dfrac{\beta}{n+\beta(n-1)c_n}+\lambda\right)c_n \end{cases}$$

This characterizes linear equilibria. The last equation is quadratic in c of the form $g(c)=0$ with

$$g(c)=\lambda\beta(n-1)(1+M_n)c^2+\left((\beta+\lambda n)(1+M_n)+(n-1)(\lambda M_n-\beta)\right)c+\left((\beta+\lambda n)\frac{M_n}{\beta}-n\right).$$

For $n-1>0$ can write it as $f(c)=c^2+\Sigma c+\Lambda=0$ where $\Sigma=\beta\Lambda+\frac{(\beta+\lambda n)+\lambda(n-1)M_n+\beta}{\beta\lambda(n-1)(1+M_n)}$ and $\Lambda=\frac{(\beta+\lambda n)M_n-\beta n}{(n-1)\lambda\beta^2(1+M_n)}$. (For n = 1 there is a unique solution to the quadratic equation.) The function $f(\cdot)$ is convex. Let $\Delta\equiv\Sigma^2-4\Lambda$. It can be checked that $\Delta>0$ and therefore the equation has two real roots and only the largest root c_n is compatible with the second order condition.

It is easily checked also that $1+\beta c_n>0$ since $f\left(-\frac{1}{\beta}\right)=\frac{-2\beta-\lambda}{\lambda\beta^2(n-1)(1+M_n)}<0$ and therefore for the largest root we have $c_n>-1/\beta$ because of convexity of $f(\cdot)$.

Furthermore, $a_n > 0$. This is so since from the expression for a_n in the system of equations, $\operatorname{sgn}(a_n) = \operatorname{sgn}\left(\frac{\beta}{n+\beta(n-1)c_n} + \lambda\right)$. Since $\beta c_n > -1$ it follows that $n+\beta(n-1)c_n > n-(n-1) = 1 > 0$. It is immediate also that the largest root decreases with M_n since $\partial f / \partial M_n > 0$. Finally, it is immediate to check from the solution to the quadratic equation that when $M_n = 0$ we have that $c_n > 0$ ♦

<u>Claim</u>: c_n^c is strictly decreasing in n if $\rho > 0$

<u>Proof</u>: We know that $c_n^c = \frac{\beta - \lambda M_n}{\lambda\beta(M_n + 1)}$. It is immediate that c_n^c is strictly decreasing in M_n and, for $\rho > 0$, we have that

$$\frac{\partial M_n}{\partial n} = \frac{\rho\sigma_\varepsilon^2}{(1-\rho)}\left(\frac{\left(\sigma_\varepsilon^2 + (1-\rho)\sigma_\theta^2\right)}{\left(\sigma_\varepsilon^2 + (1+(n-1)\rho)\sigma_\theta^2\right)^2}\right) > 0. \blacklozenge$$

In order to perform welfare comparisons we will need the following Lemma.

<u>Lemma</u>. <u>Comparison of regimes with symmetric strategies and information structure</u>. The difference in (per capita) ETS between a price-taking regime R and another regime with strategies based on less information (that is, on a weakly coarser information partition) is given by

$$\left(\text{ETS}^R \text{ - ETS}\right)/n = \left(\beta E\left[\left(\tilde{x}_n - \tilde{x}_n^R\right)^2\right] + \lambda E\left[\left(x_{in} - x_{in}^R\right)^2\right]\right)/2.$$

The result follows considering a Taylor series expansion of TS (stopping at the second term due to the quadratic nature of the payoff) around price-taking equilibria. The key to simplify the computations is to notice that at price-taking equilibria total surplus is maximized.

<u>Allocative and productive inefficiency.</u> Consider a symmetric information structure. We can decompose the total inefficiency of a regime with symmetric strategies with respect to a price-taking regime R in terms of allocative and productive inefficiency. Let $u_i \equiv x_{in} - x_n$ and $v_i \equiv x_{in}^R - x_n^R$. Then we can show that

$$\left(ETS^{R}-ETS\right)/n=\left((\beta+\lambda)E\left[\left(x_{n}-x_{n}^{R}\right)^{2}\right]+\lambda E\left[\left(u_{i}-v_{i}\right)^{2}\right]\right)/2,$$

where $\left(\beta+\lambda\right)E\left[\left(x_{n}-x_{n}^{R}\right)^{2}\right]/2$ corresponds to allocative inefficiency (loss in surplus when producing, in a cost-minimizing way, an average output x_n different from the benchmark x_n^R) and $\lambda E\left[\left(u_{i}-v_{i}\right)^{2}\right]/2$ to productive inefficiency (production of an average output in a non cost-minimizing way). The decomposition follows noting that if average outputs x_n and x_n^R are produced in a cost minimizing way then for all i, then $x_{in}-x_{in}^{R}=x_{n}-x_{n}^{R}$.

Proof of Proposition 2: Let us show first the order for the price difference $E\left[\left(p_{n}-p_{n}^{c}\right)^{2}\right]=\operatorname{var}\left[\left(p_{n}-p_{n}^{c}\right)\right]+\left(E\left[p_{n}-p_{n}^{c}\right]\right)^{2}$. We have that

$$\operatorname{var}\left[p_{n}-p_{n}^{c}\right]=\frac{\beta^{2}}{\left(1+\beta c_{n}\right)^{2}\left(1+\beta c_{n}^{c}\right)^{2}}\left(\left(a_{n}-a_{n}^{c}\right)+\beta\left(\left(a_{n}-a_{n}^{c}\right)c_{n}^{c}+a_{n}^{c}\left(c_{n}^{c}-c\right)\right)\right)^{2}\operatorname{var}\left[\tilde{s}_{n}\right],$$

and

$$\begin{aligned}E\left[p_{n}-p_{n}^{c}\right]=&\frac{1}{\left(1+\beta c_{n}\right)\left(1+\beta c_{n}^{c}\right)}\Big(\alpha\beta\left(c_{n}^{c}-c_{n}\right)+\beta\left(b_{n}^{c}-b_{n}\right)-\beta^{2}\left(c_{n}^{c}\left(b_{n}-b_{n}^{c}\right)-b_{n}^{c}\left(c_{n}-c_{n}^{c}\right)\right)\\&+\left(\beta\left(a_{n}-a_{n}^{c}\right)+\beta^{2}\left(c_{n}^{c}\left(a_{n}-a_{n}^{c}\right)-a_{n}^{c}\left(c_{n}-c_{n}^{c}\right)\right)\right)\Big)E\left[\tilde{s}_{n}\right].\end{aligned}$$

The terms $E\left[\tilde{s}_{n}\right]$ and $\operatorname{var}\left[\tilde{s}_{n}\right]$ are of the order of a constant. Therefore, the order of $E\left[\left(p_{n}-p_{n}^{c}\right)^{2}\right]$ depends only on the order of the coefficients a, b and c. We obtain:

$a_{n}-a_{n}^{c}=a\left(\frac{-\beta}{\beta+\lambda n+\lambda\beta\left(n-1\right)c_{n}}\right)$, which is of order $1/n$;

$b_{n}-b_{n}^{c}=\frac{\overline{\theta}}{\left(1+M_{n}\right)\lambda}\frac{\sigma_{\varepsilon}^{2}}{\left(\sigma_{\varepsilon}^{2}+\left(1+\left(n-1\right)\rho\right)\sigma_{\theta}^{2}\right)}\left(\frac{\beta}{\beta+\lambda n+\lambda\beta\left(n-1\right)c_{n}}\right)$, which is of order $1/n^{2}$;

$c_n - c_n^c = \left(\left(\frac{\beta}{n + \beta(n-1)c_n} + \lambda \right)^{-1} - \lambda^{-1} \right) / (M_n + 1)$, which is of order 1/n (note that M_n is of the order of a constant). We conclude that $E\left[\left(p_n - p_n^c \right)^2 \right]$ is of the order of $1/n^2$ since it is a quadratic function of the terms $\left(a_n - a_n^c\right)$, $\left(b_n - b_n^c\right)$ and $\left(c_n - c_n^c\right)$.

We deal now with the order of magnitude of $\left(ETS_n^c - ETS_n\right)/n$ using the lemma above. We have that $E\left[\left(\tilde{x}_n - \tilde{x}_n^c \right)^2 \right] = \beta^{-1} E\left[\left(p_n^c - p_n \right)^2 \right]$, since $p_n = \alpha - \beta \tilde{x}_n$ and $p_n^c = \alpha - \beta \tilde{x}_n^c$, and we know from the first part that this is of order $1/n^2$. (In fact, it is easy to see that

$$\mathrm{var}\left[\tilde{x}^c - \tilde{x} \right] = \left(\left(\frac{a_n^c}{1 + \beta c_n^c} \right) - \left(\frac{a_n}{1 + \beta c_n} \right) \right)^2 \mathrm{var}\left[\tilde{s}_n \right]$$

and the first term is of order $1/n^2$ while the second is of the order of a constant.)

Now,

$$E\left[\left(x_{in} - x_{in}^c \right)^2 \right] = \left(E\left[x_{in} - x_{in}^c \right] \right)^2 + \mathrm{var}\left[x_{in} - x_{in}^c \right]$$

and

$$E\left[x_{in} - x_{in}^c \right] = \left(b_n - b_n^c \right) - \left(a_n - a_n^c \right) \overline{\theta} + \left(c_n - c_n^c \right) E\left[p_n \right] + c_n^c E\left[p_n - p_n^c \right].$$

All the terms are of order 1/n and therefore $E\left[\left(x_{in} - x_{in}^c \right) \right]^2$ is of order $1/n^2$. We have

$$\mathrm{var}\left[x_{in} - x_{in}^c \right] = \mathrm{var}\left[\left(a_n^c - a_n \right) s_i + \left(c_n - c_n^c \right) p_n + c_n^c \left(p_n - p_n^c \right) \right].$$

With some manipulations we obtain

$$\mathrm{var}\left[\tilde{x}_i^c - \tilde{x}_i \right] = \left(a_n^c - a_n \right)^2 \mathrm{var}\left[s_i \right] + \left(\left(\left(\frac{\beta a_n^c c_n^c}{1 + \beta c_n^c} \right) - \left(\frac{\beta a_n c_n}{1 + \beta c_n} \right) \right)^2 + 2\beta \left(a_n^c - a_n \right) \left(\left(\frac{a_n c_n}{1 + \beta c_n} \right) - \left(\frac{a_n^c c_n^c}{1 + \beta c_n^c} \right) \right) \right) \mathrm{var}\left[\tilde{s}_n \right]$$

For the first summand we know that $\left(a_n^c - a_n\right)^2$ is of the order of $1/n^2$ and $\operatorname{var}\left[s_i\right]$ is a constant. For the second summand, it is immediate that the two terms in the bracket are of the order of $1/n^2$ while $\operatorname{var}\left[\tilde{s}_n\right]$ is of the order of a constant. The conclusion follows.♦

Appendix II: Cournot competition

Cournot competition

Consider the market exactly as before but now firm i sets a quantity contingent on its information $\{s_i\}$.[17] The firm has no other source of information and, in particular, does not condition on the price. The expected profits of firm i conditional on receiving signal s_i and assuming firm j, j ≠ i, uses strategy $X_j(s_j)$, are

$$E\left[\pi_i \left| s_i\right.\right] = x_i\left(P_n\left(\sum_{j\neq i} X_j(s_j) + x_i\right) - E\left[\theta_i \left| s_i\right.\right]\right) - \frac{\lambda}{2}x_i^2 .$$

From the F.O.C. of the optimization of a firm we obtain

$$p - \left(E\left[\theta_i \left| s_i\right.\right] + \lambda x_i\right) = \frac{\beta}{n} x_i .$$

(Note that given that the profit function is strictly concave and the information structure symmetric, equilibria will be symmetric.) It follows that

$$\frac{p - E\left[MC_n\right]}{p} = \frac{1}{n\eta_n}$$

where $E_n\left[MC_n\right] \equiv \frac{1}{n}\sum_{i=1}^{n}\left(E\left[\theta_i \left| s_i\right.\right] + \lambda x_i\right) = \frac{1}{n}\sum_{i=1}^{n} E\left[\theta_i \left| s_i\right.\right] + \lambda \tilde{x}_n$ and $\eta_n = p/\left(\beta \tilde{x}_n\right)$. The margins are larger or smaller than in the supply function equilibrium case depending on whether the slope of supply c_n is positive or negative since in the Cournot case they correspond to the case of $c_n = 0$.

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17 See Vives (2002) for related results when cost parameters are i.i.d. and Vives (1988) for the common value case.

The following proposition characterizes the Bayesian Cournot equilibrium and the price-taking Bayesian Cournot equilibrium (denoted by a superscript c). Both equilibria are different from their supply function counterparts (except in the knife-edge case for which $c_n = 0$) since there is no conditioning in the market price. We will abuse somewhat notation and we will use the same notation for parameters at the Cournot equilibrium than at the supply function one.

Proposition A.1. There is a unique equilibrium and a unique price-taking Bayesian Cournot equilibrium. They are symmetric, and affine in the signals. Letting $\xi \equiv \sigma_\theta^2/(\sigma_\theta^2+\sigma_\varepsilon^2)$ the strategies of the firms are given (respectively) by:

$$X_n(s_i) = b_n(\alpha-\bar{\theta}) - a_n(s_i-\bar{\theta}), \text{ where } a_n = \frac{\xi}{\frac{2\beta}{n}+\lambda+\beta\frac{n-1}{n}\rho\xi}, \text{ and } b_n = \frac{1}{\lambda+\beta\left(\frac{1+n}{n}\right)};$$

$$X_n^c(s_i) = b_n^c(\alpha-\bar{\theta}) - a_n^c(s_i-\bar{\theta}), \text{ where } a_n^c = \frac{\xi}{\frac{\beta}{n}+\lambda+\beta\frac{n-1}{n}\rho\xi}, \text{and } b_n^c = \frac{1}{\lambda+\beta}.$$

Proof: Drop the subscript n labeling the replica market and let $\beta = 1$. We consider first the Bayesian Cournot equilibrium. We check that the candidate strategies form an equilibrium. The expected profits of firm i conditional on receiving signal s_i and assuming firm j, $j \neq i$, uses strategy $X_j(\cdot)$, are

$$E\left[\pi_i|s_i\right] = x_i\left(\alpha - E\left[\theta_i|s_i\right] - \frac{1}{n}\sum_{j\neq i}E\left[X_j(s_j)|s_i\right] - \left(\frac{1}{n}+\frac{\lambda}{2}\right)x_i\right).$$

Then first order conditions (F.O.C.) yield

$$2\left(\frac{1}{n}+\frac{\lambda}{2}\right)x_i(s_i) = \alpha - E\left[\theta_i|s_i\right] - \frac{1}{n}\sum_{j\neq i}E\left[X_j(s_j)|s_i\right], \qquad \text{for } i = 1,\dots, n.$$

Plugging in the candidate equilibrium strategy and using the formulae for the conditional expectations for $E\left[\theta_i \left|s_i\right.\right]$ and $E\left[s_j \left|s_i\right.\right]$,

$$E\left[\theta_i \left|s_i\right.\right]=\xi s_i+\left(1-\xi\right)\bar{\theta} \text{ and } E\left[s_j \left|s_i\right.\right]=E\left[\theta_j \left|s_i\right.\right]=\xi\rho s_i+\left(1-\xi\rho\right)\bar{\theta},$$

it is easily checked that they satisfy the F.O.C. (which are also sufficient in our model). To prove uniqueness we show that the Bayesian Cournot equilibria of our game are in one-to-one correspondence with the (person-by-person) optimization of an appropriately defined concave quadratic team function G. A team decision rule $\left(X_1\left(s_1\right),...,X_n\left(s_n\right)\right)$ is (person-by-person) optimal if it can not be improved upon by changing only one component $X_i\left(\cdot\right)$ (i.e. each agent maximizes the team objective conditional on his information and taking as given the strategies of the other agents.) Let $G\left(x\right)=\pi_i\left(x\right)+f_i\left(x_{-i}\right)$ where

$$f_i\left(x_{-i}\right)=\sum_{j\neq i}\left(\alpha-\theta_j\right)x_j-\left(\frac{1}{n}+\frac{\lambda}{2}\right)\sum_{j\neq i}x_j^2-\frac{1}{2n}\sum_{\substack{k\neq j\\ k,j\neq i}}x_k x_j .$$

This yields

$$G\left(x\right)=\sum_j\left(\alpha-\theta_j\right)x_j-\left(\frac{1}{n}+\frac{\lambda}{2}\right)\sum_j x_j^2-\frac{1}{2n}\sum_{i\neq j}x_i x_j .$$

We obtain the same outcome by solving either $\max_{x_i} E\left[\pi_i \left|s_i\right.\right]$ or $\max_{x_i} E\left[G\left|s_i\right.\right]$ since $f_i\left(x_{-i}\right)$ does not involve x_i.

Note now that person-by-person optimization is equivalent in our context to the global optimization of the team function (since the random term does not affect the coefficients of the quadratic terms and the team function is concave in actions, Radner (1962, Theorem 4)). Invoke the result by Radner (1962, Theorem 5)), which implies that in our model, the components of the unique Bayesian team decision function of the equivalent team problem are affine. Based on the above three observations we conclude that the affine Bayesian Cournot equilibrium is the unique equilibrium.

A similar argument establishes the result for the Bayesian price-taking equilibrium. Then the F.O.C. for firm i is given by

$$\lambda X_i(s_i) = \alpha - E[\theta_i | s_i] - \frac{1}{n}\sum_j E[X_j(s_j)|s_i],$$

and the solution is a (person-by-person) maximum of a team problem with an objective function which is precisely the ETS. ♦

We consider, as before, convergence to price taking and its speed as the economy is replicated. The following proposition characterizes the convergence of the Bayesian Cournot equilibrium to a price-taking equilibrium. ETS (ETS_n^c) denotes here the expected total surplus at the (price-taking) Bayesian Cournot equilibrium.

<u>Proposition A.4</u>. As the market grows large the market price p_n at the Bayesian Cournot equilibrium converges in mean square to the price-taking Bayesian Cournot price p_n^c at the rate of $1/n$. (That is, $E\left[(p_n - p_n^c)^2\right]$ tends to 0 at the rate of $1/n^2$.) The difference $\left(ETS_n^c - ETS_n\right)/n$ is of the order of $1/n^2$.

<u>Proof</u>: Consider wlog the case $\beta = 1$. Let $y_n = p_n - p_n^c = \tilde{x}_n^c - \tilde{x}_n = (b_n^c - b_n)(\alpha - \bar{\theta}) + (a_n - a_n^c)(\tilde{s}_n - \bar{\theta})$. Recall that $E\left[(y_n)^2\right] = (E[y_n])^2 + \mathrm{var}[y_n]$. We have that $E[y_n] = (b_n^c - b_n)(\alpha - \bar{\theta})$ because $E[\tilde{s}_n] = \bar{\theta}$. It is easily seen that $(b_n^c - b_n)$ is of order $1/n$ (indeed, $n(b_n^c - b_n)$ tends to $1/(1+\lambda)^2$ as n tends to infinity). Therefore $(E[y_n])^2$ is of order $1/n^2$. Furthermore, $\mathrm{var}[y_n] = (a_n - a_n^c)^2 \mathrm{var}[\tilde{s}_n]$. We have that $\mathrm{var}[\tilde{s}_n] = \left((1+(n-1)\rho)\sigma_\theta^2 + \sigma_\varepsilon^2\right)/n$, which is

of the order of a constant for $\rho > 0$ (or $1/n$ for $\rho = 0$), and that $\left(a_n - a_n^c\right)$ is of order $1/n$ (because $n\left(a_n - a_n^c\right)$ tends to $-\xi\left(\rho\xi + \lambda\right)^{-2}$ as n tends to infinity). Therefore the order of $\operatorname{var}\left[y_n\right]$ is $1/n^2$ for $\rho > 0$ (or $1/n^3$ for $\rho = 0$). We conclude that in any case the order of $y_n = p_n - p_n^c$ is $1/n$. Consider $\left(ETS_n^c - ETS_n\right)/n$ now. According to the Lemma above and given that equilibria are symmetric we have that $\left(ETS_n^c - ETS_n\right)/n = \left(\beta E\left[\left(\tilde{x}_n - \tilde{x}_n^c\right)^2\right] + \lambda E\left[\left(\tilde{x}_{in} - \tilde{x}_{in}^c\right)^2\right]\right)/2$. We have just shown $E\left[\left(\tilde{x}_n - \tilde{x}_n^c\right)^2\right]$ to be of order $1/n^2$. We have that $E\left[\left(x_{in} - x_{in}^c\right)^2\right] = \left(E\left[x_{in} - x_{in}^c\right]\right)^2 + \operatorname{var}\left[x_{in} - x_{in}^c\right]$. Now, $E\left[x_{in} - x_{in}^c\right]$ is of the same order as $E\left[\tilde{x}_n - \tilde{x}_n^c\right]$, $1/n$, and $\operatorname{var}\left[x_{in} - x_{in}^c\right] = \left(a_n^c - a_n\right)^2\left(\sigma_\theta^2 + \sigma_\varepsilon^2\right)$, is of order $1/n^2$ because $\left(a_n^c - a_n\right)$ is of order $1/n$. Therefore, $E\left[\left(x_{in} - x_{in}^c\right)^2\right]$ is of order $1/n^2$. We conclude that $\left(ETS_n^c - ETS_n\right)/n$ is of the order of $1/n^2$. ♦

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